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Adaptive Progressive Censoring in Reliability and Survival Analysis: Methods, Design, and Practice

Mazen Nassar[1,2]

[1]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
[2]Department of Statistics, Faculty of Commerce, Zagazig University, Zagazig 44519 , Egypt

Mazen Nassar
mmohamad3@kau.edu.sa

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Abstract

Incomplete lifetime data frequently arise in reliability and survival experiments due to constraints in cost, time, and operational conditions. To address these challenges, several advanced censoring schemes have been developed to enhance testing efficiency and improve the accuracy of statistical inference. This review provides a unified framework for adaptive progressive censoring and its extensions, including adaptive progressive Type-II hybrid censoring, adaptive progressive Type-I hybrid censoring, adaptive general progressive Type-II hybrid censoring, improved adaptive progressive Type-II hybrid censoring, and adaptive progressive first-failure censoring schemes. The paper summarizes their design methodologies, likelihood and spacings-based formulations, and the main inferential approaches under classical, Bayesian, and expected Bayesian paradigms. Applications involving multiple causes of failure, accelerated life testing experiments, block and joint censoring structures, and random removals are also discussed. The review further highlights the practical balance among experimental duration, cost, and estimation precision, and identifies several promising directions for future research, such as dependent competing risks, optimal censoring design, and computational developments for modern reliability and survival analysis, that may be of interest to investigators seeking further advancements in this area.

1 Introduction

In modern reliability and survival analysis, complete observation of all lifetimes is rarely achievable because testing is costly and time is limited. Many units remain functioning when resources are exhausted. Censoring is therefore essential: it preserves partial information from these units and enables valid inference without extending the study beyond feasible cost and time limits. Two classical censoring plans are widely used, namely Type-I and Type-II censoring plans. In a Type-I censoring plan (time-censored scheme), the study stops at a fixed time T determined by the testing plan. All failures before T are observed; units still operating at T are right-censored at T. The Type-I censoring plan is straightforward to implement and control, but when failures are infrequent, few events may occur before T, leading to wide uncertainty. In a Type-II censoring plan (failure-censored scheme), observation continues until the m-th failure among n units. The first m failure times are fully observed, and the remaining $n-m$ units are right-censored at the time of the m-th failure. The Type-II censoring plan ensures a predetermined number of events, thereby enhancing estimability. However, the duration of the test remains variable and may extend significantly

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in instances of slow failure rates, which in turn escalates costs and constrains throughput, see for more details Balakrishnan and Kundu [1]. Progressive Type-II censoring (PT-IIC) scheme addresses this gap by pre-scheduling the removal of surviving units following each observed failure, thereby managing test burden while simultaneously extracting information from multiple order statistics. This concept originates from the foundational work of Cohen [2] and has since been systematically developed in various monographs and numerous methodological papers, see for example Balakrishnan and Sandhu [3], Balakrishnan and Aggarwala [4], Pakyari and Balakrishnan [5], and Lin et al. [6], among others.

The PT-IIC plan begins with n units on test and a target of observing exactly m failures. Before testing, a removal pattern $\mathbf{R} = (R_1, \ldots, R_m)$ is fixed, where R_i denotes the number of surviving units to be randomly withdrawn immediately after the i-th failure time $X_{i:m:n}$; feasibility requires $m + \sum_{i=1}^{m} R_i \leq n$. At each failure time $X_{i:m:n}$ $(i = 1, \ldots, m)$, the failure is recorded and R_i survivors are removed without observing their lifetimes; after the m-th failure at $X_{m:m:n}$, any remaining units are right-censored at $X_{m:m:n}$. This plan preserves the desirable feature of Type-II censoring, namely, a guaranteed number of failures for inferential stability, while reducing the number of units carried forward, thereby decreasing expected test time and cost. For a lifetime model characterized by a probability density function (PDF) $f(x;\boldsymbol{\theta})$ and a cumulative distribution function (CDF) $F(x;\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ denotes the vector of unknown parameters, the likelihood function (LF) based on the observed failures $\mathbf{x} = (x_1, \cdots, x_m)$ (with x_i representing the realization of the $X_{i:m:n}$ order statistic) and removals $\mathbf{R}$, subject to a free combinatorial constant, is expressed as follows:

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{m} f(x_i;\boldsymbol{\theta})[1 - F(x_i;\boldsymbol{\theta})]^{R_i}. \tag{1}$$

The PT-IIC scheme has been studied and implemented across the most widely used lifetime models, utilizing a consistent analytical toolkit that includes maximum likelihood estimation, bootstrap confidence intervals, and Bayesian inference through Markov chain Monte Carlo (MCMC) methods, which also entails the computation of highest posterior density (HPD) intervals. See for example, Balakrishnan et al. [7], Raqab et al. [8], Krishna and Kumar [9], and Dey and Al-Mosawi [10]. As an alternative to the maximum likelihood estimation method, Ng et al. [11] explored the application of the maximum product of spacings estimation method for the estimation of the three-parameter Weibull distribution using the PT-IIC plan. The theoretical foundation of this method is rooted in the probability integral transform, which posits that a set of independent random samples derived from any random variable should, on average, exhibit a uniform distribution with respect to the CDF of that variable. The estimators in this context are obtained by maximizing the following spacings function (SF) with respect to the model parameters.

$$\mathcal{P}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{m+1}[F(x_i;\boldsymbol{\theta}) - F(x_{i-1};\boldsymbol{\theta})] \prod_{i=1}^{m}[1 - F(x_i;\boldsymbol{\theta})]^{R_i}, \tag{2}$$

where $F(x_0;\boldsymbol{\theta}) \equiv 0$ and $F(x_{m+1};\boldsymbol{\theta}) \equiv 1$.

Kundu and Joarder [12] introduced a Progressive Type-I hybrid censoring (PT-IHC) plan that mixtures the Type-I censoring plan with the PT-IIC scheme. In this design, n units are subjected to testing with a predetermined target of m observed failures and a fixed time cap $T > 0$; a removal pattern $\mathbf{R} = (R_1, \ldots, R_m)$ is established in advance. At each failure time $X_{i:m:n}$, where $i = 1, \ldots, m$, the failure is recorded and R_i survivors are removed without observing their lifetimes. We will have the termination time of the experiment to be $T^{\star} = \min(X_{m:m:n}, T)$, where the experiment concludes at the m-th failure $X_{m:m:n}$ if it occurs prior to T (yielding the PT-IIC outcome with m failures); otherwise, it concludes at T. However, a limitation of the PT-IHC plan, akin to traditional Type-I censoring (time censoring), is that the effective sample size is random and may ultimately be quite small (potentially even zero). Consequently, standard statistical inference procedures may not be applicable or may exhibit low efficiency. To address this limitation, Ng et al. [13] proposed an adaptive progressive Type-II hybrid censoring (APT-IIHC) scheme, which is a hybrid of the Type-I and PT-IIC methodologies. In the APT-IIHC approach, the effective number of observed failures m is predetermined, and an initial progressive removal schedule is established. However, the removal counts at each failure, denoted as $\mathbf{R} = (R_1, R_2, \ldots, R_m)$, are permitted

to adapt during the experiment. This adaptive flexibility maintains the Type-II characteristic of ensuring m failures while dynamically managing the survivor carry-over, thereby reducing total test time and failure-induced costs and enhancing inferential efficiency compared to the non-adaptive PT-IIC. The APT-IIHC scheme will be the focus of discussion in Section 2.

Since the work of Ng et al. [13], numerous adaptive censoring plans have been introduced in the literature, each with its own merits, aimed at addressing practical issues inherent in existing censoring strategies. Lin and Huang [14] proposed an adaptive progressive Type-I hybrid censoring (APT-IHC) plan and investigated the inferential challenges associated with the exponential distribution. This scheme will be reviewed in Section 3. Abu-Moussa et al. [15] introduced an adaptive-general PT-IIC (AGPT-IIC) scheme. This scheme represents a hybridization of the APT-IIHC and general PT-IIC schemes. Section 4 discusses and reviews this scheme. Yan et al. [16] introduced an enhanced version of the APT-IIHC scheme, termed the improved APT-IIHC (IAPT-IIHC) plan, which is particularly suitable for scenarios involving highly reliable tested items. Lee and Lee [17] proposed the same design under the name generalized adaptive progressive hybrid censoring (GAPHC), however, this terminology has experienced relatively limited adoption within the literature. Section 5 presents a comprehensive description of the IAPT-IIHC scheme, along with an analysis of existing studies that have utilized this framework. Recently, Fathi [18] introduced a novel adaptive censoring plan known as the adaptive progressive first-failure censoring (APFFC) scheme and examined the statistical inference related to the Weibull distribution. This scheme will be discussed in Section 6.

To the best of our knowledge, no existing study has provided a comprehensive review of all adaptive progressive censoring plans within a single work. The main objective of the present study is therefore to introduce a unified guideline and conceptual framework for these available plans, while also outlining several potential future research directions for each scheme to motivate further investigation. This review brings adaptive censoring plans together in one clear, consistent framework. It uses the same notation for removal patterns and stopping rules, compares how each plan works, and defines the LF for classical inference and the SF used in maximum product of spacings estimation. For every plan, it summarizes the key studies and offers short, ready-to-use implementation templates that can be adapted to common lifetime models. By pulling scattered results into one place, the review works as both a simple methodological guide and a practical design manual for reliability and survival studies with adaptive censoring. Finally, the conclusion is given in Section 7.

2 APT-IIHC Scheme

This section elaborates on the APT-IIHC plan, encompassing the design, schematic representation, and the corresponding likelihood and spacings formulations. Subsequently, we review the lifetime models utilized, delineate the simulation settings, and present commonly used real-data examples, along with extensions to competing risks and accelerated life tests (ALTs).

2.1 Design, likelihood, and spacings formulations

Let n identical units be placed on test, and fix an integer $m < n$ together with a progressive censoring pattern $\mathbf{R} = (R_1, \ldots, R_m)$. An ideal total test time $T > 0$ is specified, but the experiment may continue beyond T. Under the APT-IIHC plan, at the first failure time $X_{1:m:n}$, randomly withdraw R_1 of the surviving units; at the second failure time $X_{2:m:n}$, withdraw R_2 survivors; proceed similarly at $X_{i:m:n}$ by withdrawing R_i survivors for $i = 3, \ldots, m$. In this case, we have two termination scenarios:

1. **Case I:** If $X_{m:m:n} < T$, the experiment stops at $X_{m:m:n}$ and the original removal pattern $\mathbf{R}$ is applied. This is the usual PT-IIC case.
2. **Case II:** If $X_{m:m:n} > T$, let d be the number of failures observed by time T. Modify the removal

schedule to

$$R_i = \begin{cases} R_i, & i = 1, \ldots, d, \\ 0, & i = d+1, \ldots, m-1, \\ R_m^\star = n - m - \sum_{i=1}^{d} R_i \geq 0, & i = m, \end{cases}$$

that is, all scheduled removals after the d-th failure are suspended until the m-th failure, at which time the remaining survivors are withdrawn. The test then terminates at $X_{m:m:n}$.

A schematic representation of the two termination scenarios is shown in Figure 1. One can see that the APT-IIHC design targets m observed failures for inferential efficiency while keeping the total time on test close to the ideal duration T. Ng et al. [13] studied point and interval estimation of the failure rate for exponentially distributed failure times using the APT-IIHC scheme. The various methods are compared using Monte Carlo simulation. Let $\mathbf{x}$ be an observed APT-IIHC sample selected from a continuous population with PDF $f(x;\boldsymbol{\theta})$ and CDF $F(x;\boldsymbol{\theta})$. Then, the LF of the observed data can be expressed without the constant term as follows

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{m} f(x_i;\boldsymbol{\theta}) \prod_{i=1}^{d} [1 - F(x_i;\boldsymbol{\theta})]^{R_i} [1 - F(x_m;\boldsymbol{\theta})]^{R_m^\star}. \tag{3}$$

Almetwally et al. [19] introduced and studied maximum product of spacings estimation for the generalized Rayleigh distribution, while Almetwally et al. [20] investigated both likelihood-based and maximum product of spacings estimators for the Weibull distribution; see also Elshahhat and Mohammed [21] and Dutta et al. [22]. Using an observed APT-IIHC sample $\mathbf{x}$, Almetwally et al. [19] defined the SF in this case follows:

$$\mathcal{P}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{m+1} [F(x_i;\boldsymbol{\theta}) - F(x_{i-1};\boldsymbol{\theta})] \prod_{i=1}^{d} [1 - F(x_i;\boldsymbol{\theta})]^{R_i} [1 - F(x_m;\boldsymbol{\theta})]^{R_m^\star}. \tag{4}$$

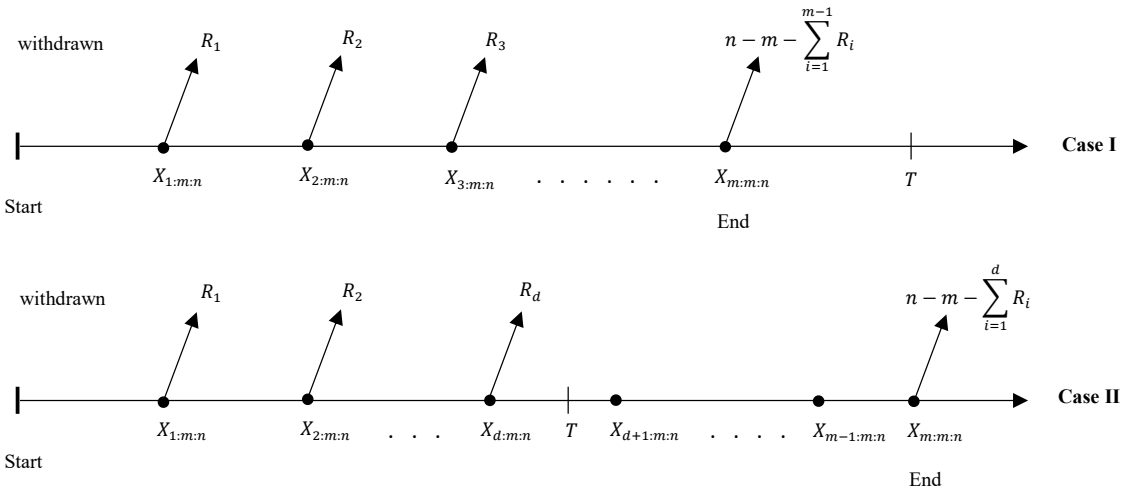

Figure 1: Schematic representation of APT-IIHC scheme.

2.2 Models under APT-IIHC: simulation and data

Numerous studies have investigated estimation procedures for a wide range of lifetime distributions under the APT-IIHC scheme. Lin et al. [23] investigated both classical maximum likelihood estimators (MLEs) and approximate MLEs for the Weibull distribution under the PT-IHC and APT-IIHC schemes.

Their results indicate that, when neither total test time nor the number of failed items is a binding constraint, the APT-IIHC is preferable because it generally delivers more precise parameter estimates. Conversely, when the objective is to shorten the experiment and/or limit the number of units that fail, the PT-IHC offers a practical alternative aligned with those operational goals. Hemmati and Khorram [24] investigated the MLEs and approximate MLEs of the log-normal distribution under both PT-IHC and APT-IIHC schemes. The study demonstrated that the performance of the APT-IIHC scheme is more satisfactory in comparison to that of the PT-IHC scheme, particularly when the pre-fixed time T is small. Ye et al. [25] examined classical estimators for the extreme value distribution and developed several interval estimation procedures. Al Sobhi and Soliman [26] investigated estimation for the exponentiated Weibull model and its reliability measures from both classical and Bayesian perspectives. Nassar et al. [27] analyzed classical and Bayesian estimation for the Weibull distribution using symmetric and asymmetric loss functions under independent gamma priors.

The literature covers a wide range of lifetime models under the APT-IIHC plan: Mahmoud et al. [28] studied the generalized Pareto distribution; Nassar and Abo-Kasem [29] analyzed the inverse Weibull model; Ateya and Mohammed [30] considered the exponentiated exponential distribution; Chen and Gui [31] investigated the truncated normal distribution; Chen and Gui [32] examined a bathtub-shaped lifetime model; Elshahhat and Nassar [33] presented a Bayesian survival analysis for Hjorth data; Mohan and Chacko [34] treated the Kumaraswamy–exponential model; Kohansal and Bakouch [35] addressed the Kumaraswamy distribution; Alotaibi et al. [36] handled the alpha-power exponential model; Yu et al. [37] provided the generalized inverted exponential model; Alotaibi et al. [38] analyzed the inverse Lindley distribution; Wang and Gui [39] investigated the exponentiated Pareto distribution; and Lv et al. [40] derived inference for the Gompertz distribution.

Together, these works illustrate how estimation procedures, classical and Bayesian, can be adapted across models while retaining the common likelihood structure induced by the APT-IIHC plan. These works tried to compare various point and interval estimation techniques using simulation study based on some accuracy criteria, including root mean square error, bias, interval length, and coverage probability. These studies considered simulation by generating large number of APT-IIHC samples using the algorithm proposed by Ng et al. [13], using common progressive censoring patterns $\mathbf{R}$, for example

$$\begin{aligned} \text{Scheme-I}: &\quad R_1 = \cdots = R_{m-1} = 0 \text{ and } R_m = n - m, \\ \text{Scheme-II}: &\quad R_1 = n - m \text{ and } R_2 \cdots = R_m = 0, \\ \text{Scheme-III}: &\quad R_1 = \cdots = R_m = (n-m)/m. \end{aligned}$$

Most studies extend beyond simulations to analyze real datasets for estimating model parameters and, in many cases, for deriving key reliability and survival measures of the fitted models. Frequently used examples include: time to breakdown of an insulating fluid between electrodes (Nelson [41]); lifetimes of sodium–sulfur batteries (Ansell and Ansell [42]); failure times of electronic devices (Wang [43]); service times of aircraft windshields (Murthy et al. [44]); and breaking stress of carbon fibres (Nichols and Padgett [45]), among others. Finally, several of the reviewed studies investigated optimality criteria to determine an optimal censoring plan.

2.3 APT-IIHC with competing risks

In medical and reliability studies, failures can arise from multiple causes, and investigators are often interested in quantifying the risk attributable to a particular cause in the presence of others. The resulting data typically comprise a failure time and an indicator of the cause of failure, this is the competing risks setting. General introductions and inferential tools are well documented by Crowder [46]. Although one may allow dependence among causes, identifiability becomes problematic in the absence of covariates, which motivates the widely used latent failure time formulation that assumes mutually independent cause-specific lifetimes; this framework dates back to Cox [47] and remains the standard starting point for model development and inference. Hemmati and Khorram [48] analyzed the APT-IIHC plan under a competing-risks framework, assuming that cause-specific lifetimes are mutually independent and follow

exponential distributions. They examined only two causes of failures, operating under the assumption that certain causes of failure remain unidentified.

Let X_{ai} denote the latent failure time of unit i due to cause $a \in \{1, 2\}$, and assume X_{1i} and X_{2i} are independent. The observed lifetime and cause indicator are

$$X_i = \min\{X_{1i},\ X_{2i}\}.$$

Under the APT-IIHC plan, the data can be written as

$$(X_{1:m:n}, \phi_1, R_1), \ldots, (X_{d:m:n}, \phi_d, R_d), (X_{d+1:m:n}, \phi_{d+1}, 0), \ldots, (X_{m-1:m:n}, \phi_{m-1}, 0), (X_{m:m:n}, \phi_m, R_m^{\star}),$$

where $\phi_i \in \{1, 2, \star\}$. Here, $\phi_i = a$ ($a \in \{1, 2\}$) indicates that unit i failed at time $X_{i:m:n}$ due to cause a, whereas $\phi_i = \star$ indicates that the cause of failure for unit i is unknown. The LF of the observed data is given by

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\theta}; \mathbf{x}) &= \prod_{i=1}^{m} \Big\{ \big[f_1(x_i;\boldsymbol{\theta})\bar{F}_2(x_i;\boldsymbol{\theta})\big]^{\mathbb{I}(\phi_i=1)} \big[f_2(x_i;\boldsymbol{\theta})\bar{F}_1(x_i;\boldsymbol{\theta})\big]^{\mathbb{I}(\phi_i=2)} \\
&\times \big[f_1(x_i;\boldsymbol{\theta}_1)\bar{F}_2(x_i;\boldsymbol{\theta}) + f_2(x_i;\boldsymbol{\theta})\bar{F}_1(x_i;\boldsymbol{\theta})\big]^{\mathbb{I}(\phi_i=\star)} \Big\} \prod_{i=1}^{d} \big[\bar{F}_1(x_i;\boldsymbol{\theta})\, \bar{F}_2(x_i;\boldsymbol{\theta})\big]^{R_i} \\
&\times \big[\bar{F}_1(x_m;\boldsymbol{\theta})\, \bar{F}_2(x_m;\boldsymbol{\theta})\big]^{R_m^{\star}}, \qquad (5)
\end{aligned}$$

where $f_a(x_i;\boldsymbol{\theta})$ and $\bar{F}_a(x_i;\boldsymbol{\theta}) = 1 - F_a(x_i;\boldsymbol{\theta})$ are the PDF and reliability functions of X_a. In such setting, we have

$$m_1 = \sum_{i=1}^{m} \mathbb{I}(\phi_i = 1),\ m_2 = \sum_{i=1}^{m} \mathbb{I}(\phi_i = 2),\ \text{and}\ m_3 = \sum_{i=1}^{m} \mathbb{I}(\phi_i = \star),$$

denote the numbers of failures due to causes 1 and 2, and the number of failures with unknown cause, respectively; with $m_1 + m_2 + m_3 = m$. In the majority of available studies, authors assumed that the cause of failure is always known, i.e., $m_3 = 0$. In this case, the likelihood in (5) reduces to

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\theta}; \mathbf{x}) &= \prod_{i=1}^{m} \Big\{ \big[f_1(x_i;\boldsymbol{\theta})\bar{F}_2(x_i;\boldsymbol{\theta})\big]^{\mathbb{I}(\phi_i=1)} \big[f_2(x_i;\boldsymbol{\theta})\bar{F}_1(x_i;\boldsymbol{\theta})\big]^{\mathbb{I}(\phi_i=2)} \Big\} \\
&\times \prod_{i=1}^{d} \big[\bar{F}_1(x_i;\boldsymbol{\theta})\, \bar{F}_2(x_i;\boldsymbol{\theta})\big]^{R_i} \big[\bar{F}_1(x_m;\boldsymbol{\theta})\, \bar{F}_2(x_m;\boldsymbol{\theta})\big]^{R_m^{\star}}. \qquad (6)
\end{aligned}$$

Many studies have examined classical and Bayesian estimation for a range of lifetime models under the APT-IIHC scheme with independent competing risks. For example, Liu and Gui [49] analyzed the two-parameter Rayleigh model; Nassr et al. [50] considered the extended Weibull distribution; Nassar et al. [51] investigated estimation for the Rayleigh distribution; Du and Gui [52] studied the Burr XII model; Almuqrin et al. [53] developed methods for the Gompertz distribution; Qin and Gui [54] treated the Lomax distribution; and Hasaballah et al. [55] analyzed the generalized logistic distribution, among others.

Most existing studies assume independent causes of failure, an assumption that is often unrealistic in practice. In clinical settings, for example, colon cancer, multiple factors such as cancer recurrence, declining physiological function, and impaired immunity can interact, making independence untenable. It is therefore more appropriate in many real-world applications to model dependent causes of failure. For analyzing data under dependent competing-risks structures, Marshall–Olkin bivariate (MOB) distributions are frequently employed; see Samanta and Kundu [56] for details. In a two-cause competing-risks framework, let X_1 and X_2 represent the lifetimes associated with causes 1 and 2, respectively. The random variable $X = \min(X_1, X_2)$ denotes the lifetime of an experimental unit exposed to both risk factors. In addition to the failure time, the specific cause of failure is also documented. Under the MOB distributions

framework, failure may occur due to cause 1, cause 2, or concurrently from both causes, thereby providing a realistic representation of dependent risks.

Under the APT-IIHC scheme, relatively few studies have addressed estimation for lifetime models with dependent competing risks. Notable exceptions include Du and Gui [57], who developed inference for the MOB exponential model; Vardani et al. [58], who investigated the MOB Kumaraswamy distribution; Dutta and Kayal [59], who analyzed the MOB Weibull distribution; and Zhang et al. [60], who proposed Bayesian methods for the MOB Lomax–geometric distribution.

2.4 APT-IIHC with ALTs

In many applications, lifetimes at use conditions are hard to observe because products are highly reliable. Reliability practitioners therefore employ ALTs, increasing one or more stresses, e.g., temperature, voltage, load, humidity, to induce earlier failures and obtain information in a shorter time. Common ALT designs include constant-stress ALTs, where each unit is tested at a fixed elevated stress; step-stress ALTs, where stress is increased in stages during a unit's life; and progressive-stress ALTs, with stress changing more continuously or in many small steps. Typical life–stress links pair these designs with models such as Arrhenius/Eyring (temperature/chemical), inverse-power (electrical or mechanical stress), and log-linear parametrizations, enabling extrapolation from accelerated to normal use while quantifying uncertainty.

Several studies have analyzed the APT-IIHC plan in conjunction with ALTs. For constant-stress ALTs, see Guangyu and Yimin [61] for the generalized exponential model, Zhang et al. [62] for dependent competing risks via a bivariate Birnbaum–Saunders distribution, and Cui et al. [63] for the Weibull distribution. Research on step-stress ALTs under APT-IIHC is limited; a notable contribution is Zhang [64], who developed classical estimation for Weibull parameters. For progressive-stress ALTs, the literature is sparse; to our knowledge only two studies exist: Hashem et al. [65] for Poisson–logarithmic half-logistic distribution, and Hussam and Almetwally for Perk distribution. On the other hand, step-stress partially ALTs have attracted considerable attention. Representative studies include Ismail [67] for the Weibull distribution, Nassar et al. [68] for Burr-XII distribution, Kamal et al. [69] for Nadarajah–Haghighi distribution, Alotaibi et al. [70] for alpha power inverse Weibull distribution, and Alam and Ahmed [71] for the exponentiated Pareto model. For the constant-stress partially ALTs studies, we come across only two studies for Abd-Elmougod and Mahmoud [72] regarding Rayleigh distribution, and Weibull inverted exponential distribution by Ahmed [73]. For constant-stress partially ALTs, we identified only two studies: Abd-Elmougod and Mahmoud [72] for the Rayleigh distribution, and Ahmed [73] for the Weibull inverted exponential distribution.

Substantial open problems persist for the APT-IIHC scheme under ALTs, particularly concerning step-stress ALTs and constant-stress partially ALTs. Key priorities include the development of rigorous estimation procedures for widely utilized models, such as Weibull and log-normal, accompanied by point and interval estimation, reliability assessments, and design guidance.

2.5 APT-IIHC Design Extensions

This subsection surveys extensions that enhance the scope and applicability of the APT-IIHC scheme. We conduct a review of random removals, joint and block APT-IIHC designs for multi-batch or multi-line testing.

APT-IIHC scheme with random removals

In the standard APT-IIHC plan, the removal counts $R_1, \ldots, R_m$ are fixed in advance. In practice, that's often unrealistic: the number of units that leave the study after each failure can vary unpredictably, for example, patient dropouts in a trial or rigs temporarily reassigned in a lab. A simple fix is to treat each removal R_i as random rather than fixed. For the group still at risk just before the i-th failure, we can choose a reasonable discrete model for R_i. For the conventional PT-IIC scheme, Yuen and Tse [74] modeled the number of removed items using a discrete uniform distribution, whereas Tse et al. [75]

argued that a binomial distribution provides a more realistic specification. This idea has been extended to the APT-IIHC setting by treating the number of removals at each stage as a random variable following a binomial distribution. Elshahhat and Nassar [76] developed this formulation and conducted both classical and Bayesian inference for the Weibull life model, estimating not only the Weibull parameters but also the binomial parameter. In a related contribution, Alqasem et al. [77] analyzed the Burr–X competing risks model with binomial removals, further illustrating how random withdrawal mechanisms can be embedded within the APT-IIHC design. See also the work of Alotaibi et al. [78] for the inverse Chen distribution.

One important open problem remains in this context. Another flexible discrete model capable of modeling the removal pattern, such as the beta-binomial distribution, requires further investigation to assess its effects on reliability estimation in comparison to binomial removals.

Joint APT-IIHC scheme

There are instances in which the researcher seeks to compare multiple populations simultaneously. In such scenarios, joint censoring schemes may be employed. Rather than analyzing each group with a distinct censoring mechanism, all groups are observed at the same time criterion, resulting in jointly censored samples. This approach is particularly relevant in reliability and survival studies, as tests are conducted in parallel to optimize cost and time efficiency while facilitating direct comparisons across different designs, materials, and treatments. Sultana et al. [79] introduced a joint APT-IIHC scheme for independent samples drawn from two populations and developed inference for the exponential distribution. Subsequent work has extended the joint framework to other models, including the Weibull distribution by Sultana et al. [80], the inverse Weibull distribution by Al-Essa et al. [81], the generalized Lindley distribution by Anakha and Chacko [82], and the inverted exponentiated family by Kumari et al. [83].

An important issue in this context is that the existing studies have predominantly concentrated on the joint APT-IIHC scheme involving only two populations. While this assumption simplifies the analysis, it can be extended to incorporate multiple populations for greater practical applicability.

Block APT-IIHC scheme

In many studies, it is not feasible to monitor all units simultaneously, owing to limited or hard-to-access test equipment and other practical constraints that prevent placing every unit on test at once. To accommodate these realities, Ahmadi et al. [84] introduced a more flexible block censoring scheme, in which units are tested in batches; this design aligns with resource limitations and can substantially improve test efficiency. The scheme proceeds as follows: let n_i $(i = 1, 2, \ldots, k)$ denote the number of units in the i-th group, with $\sum_{i=1}^{k} n_i = n$ identical test units allocated across k groups. The k groups are tested in parallel using k facilities, each operated under Type-II censoring conditions. Recently, this setting has attracted considerable attention as a way to enhance the flexibility of existing censoring schemes. Among these, the block APT-IIHC scheme is a notable example. Singh et al. [85] analyzed the block APT-IIHC scheme and developed classical estimation methods for the Weibull distribution. Nevertheless, the literature in this area remains limited and would benefit from further investigation, including applications to additional lifetime models, extensions to competing risks frameworks, and integration with ALTs.

3 APT-IHC Scheme

The APT-IIHC scheme guarantees termination upon observing a pre-specified number of failures, denoted as m. However, in contemporary products characterized by compressed development cycles and stringent time constraints, reliability testing frequently cannot await the occurrence of m failures. Under these severe time limitations, failure-censored plans such as APT-IIHC plan may prove impractical for numerous fielded products, thereby necessitating the consideration of time-censored or hybrid designs with fixed time limitations. To address this limitation, Lin and Huang [14] proposed the APT-IHC scheme, which guarantees termination at a fixed time T and, in practice, yields higher estimation efficiency than the

PT-IHC scheme. This section focuses on the APT-IHC scheme, its design and the associated inferential methods.

3.1 Design, likelihood, and spacings formulation

Suppose n identical units are placed on test with prefixed removal pattern $\mathbf{R} = (R_1, R_2, \ldots, R_m)$ with $1 \leq m \leq n$, and the experiment is scheduled to terminate at a fixed time $T \in (0, \infty)$. At the first failure time $X_{1:m:n}$, randomly withdraw R_1 of the surviving units; at the second failure time $X_{2:m:n}$, withdraw R_2 survivors; and so on. Let $d = \max\{\, i \in \{0, 1, \ldots, m\} : X_{i:m:n} \leq T \,\}$ denote the number of failures that occur no later than time T. Then, we can observe one of the following cases:

1. **Case I**: If $X_{m:m:n} > T$, the test terminates at time T; withdrawals are applied only up to the last failure observed no later than T, yielding the realize progressive censoring scheme $(R_1, R_2, \ldots, R_d)$.
2. **Case II**: If $X_{m:m:n} < T$ the process does not stop at $X_{m:m:n}$; instead, observations continue with no further withdrawals until time T. At T, all remaining units are removed and the test terminates. The number removed at T is $n - d - \sum_{i=1}^{d} R_i$, and the realize progressive censoring scheme is $(R_1, R_2, \ldots, R_{m-1}, 0, \ldots, 0, R_d)$.

Let the resulting failure times be $X_{1:m:n}$, $X_{2:m:n}$, $\ldots$, $X_{m:m:n}$, $X_{m+1:n}$, $\ldots$, $X_{d:n}$ when $X_{m:m:n} < T$, and $X_{1:m:n}$, $X_{2:m:n}$, $\ldots$, $X_{d:m:n}$ when $X_{m:m:n} > T$. A schematic representation of the two APT-IHC termination scenarios is shown in Figure 2. Based on an observed APT-IHC sample, say $\mathbf{x}$, collected from a continuous population with PDF $f(x;\boldsymbol{\theta})$ and CDF $F(x;\boldsymbol{\theta})$. The LF can be expressed without the constant term as follows

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{d} f(x_i;\boldsymbol{\theta})[1 - F(x_i;\boldsymbol{\theta})]^{R_i}[1 - F(T;\boldsymbol{\theta})]^{R_d^\star}, \tag{7}$$

where $R_d^\star$ denote the number of removals at time T as displayed in Figure 2. Using the same setting, Nassar et al. [86] developed the SF under the APT-IHC scheme as follows

$$\mathcal{P}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{d+1}[F(x_i;\boldsymbol{\theta}) - F(x_{i-1};\boldsymbol{\theta})] \prod_{i=1}^{d}[1 - F(x_i;\boldsymbol{\theta})]^{R_i}[1 - F(T;\boldsymbol{\theta})]^{R_d^\star}. \tag{8}$$

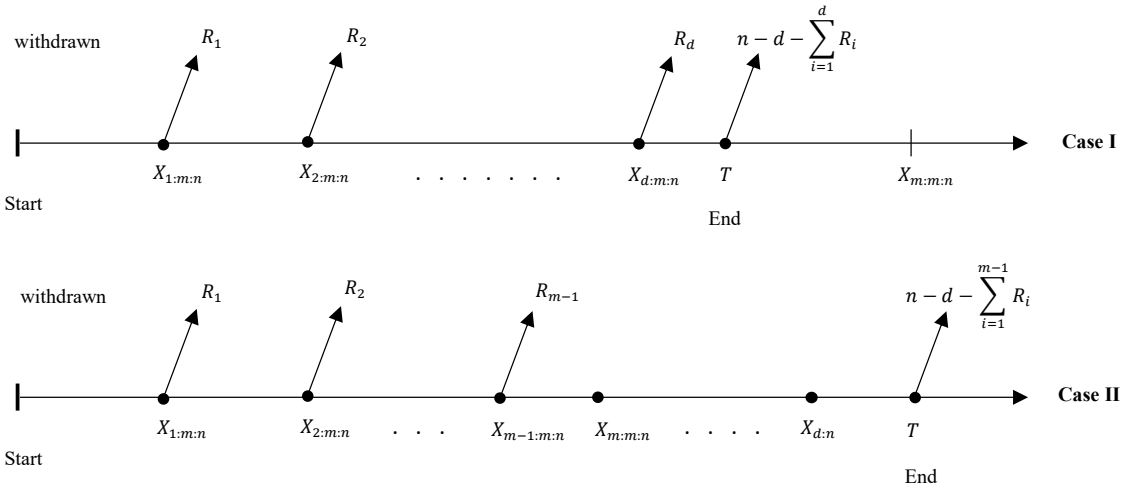

Figure 2: Schematic representation of APT-IHC scheme.

3.2 Inference based on APT-IHC scheme

Several studies have developed inference for a range of lifetime models under the APT-IHC scheme, encompassing single-cause settings, competing risks frameworks, and integrations with ALTs. These

contributions address parameter estimation, interval procedures, and design considerations across the different contexts.

Lin and Huang [14] investigated the inference of the mean lifetime of an exponential distribution, encompassing various methods of interval estimation. Lin et al. [87] examined both classical and Bayesian estimations of the parameters of the Weibull distribution. Additionally, they proposed a cost-based procedure to determine the optimality of the APT-IHC plan. Nassar and Dobbah [88] conducted classical and Bayesian inference for a bathtub-shaped lifetime distribution, estimating model parameters as well as reliability and hazard rate measures. Okasha et al. [89] developed classical, Bayesian, and E-Bayesian estimators for the reliability and hazard rate functions of the Lomax distribution. Almarashi et al. [90] developed reliability estimation procedures for the Nadarajah–Haghighi distribution, deriving Bayes estimates via Lindley's approximation and MCMC procedure. Abo-Kasem et al. [91] investigated parameter estimation for the inverted Nadarajah–Haghighi distribution. A clear gap under the APT-IHC scheme is the selection of an optimal progressive censoring plan; systematic criteria and robust design algorithms remain underdeveloped and warrant further investigation.

In the context of competing risks, Ashour and Nassar [92] studied the inference of Weibull competing risks model. Using the competing-risks notation introduced earlier, the APT-IHC data can be written as $(X_{1:m:n}, \phi_1, R_1), \ldots, (X_{m-1:m:n}, \phi_{m-1}, R_{m-1}), (X_{m:n}, \phi_m, 0), \ldots, (X_{d:n}, \phi_d, 0)$ when $X_{m:m:n} < T$, and $(X_{1:m:n}, \phi_1, R_1), (X_{2:m:n}, \phi_2, R_2), \ldots, (X_{d:m:n}, \phi_d, R_d)$ when $X_{m:m:n} > T$. They defined the LF of the observed data as follows

$$\begin{aligned} \mathcal{L}(\boldsymbol{\theta}; \mathbf{x}) &= \prod_{i=1}^{d} \left[f_1(x_i;\boldsymbol{\theta}) \bar{F}_2(x_i;\boldsymbol{\theta}) \right]^{\mathbb{I}(\phi_i=1)} \left[f_2(x_i;\boldsymbol{\theta}) \bar{F}_1(x_i;\boldsymbol{\theta}) \right]^{\mathbb{I}(\phi_i=2)} \left[\bar{F}_1(x_i;\boldsymbol{\theta})\, \bar{F}_2(x_i;\boldsymbol{\theta}) \right]^{R_i} \\ &\times \left[\bar{F}_1(T;\boldsymbol{\theta})\, \bar{F}_2(T;\boldsymbol{\theta}) \right]^{R_d^\star}, \end{aligned} \tag{9}$$

where $d_1 = \sum_{i=1}^{d} \mathbb{I}(\phi_i = 1)$ and $d_2 = \sum_{i=1}^{d} \mathbb{I}(\phi_i = 2)$ denote the numbers of failures due to the first and second causes, respectively, with $d_1 + d_2 = d$. Other studies have analyzed competing-risks models under the APT-IHC scheme for various lifetime distributions. Okasha and Mustafa [93] developed E-Bayesian estimation for the Weibull model; Abu El Azm et al. [94] examined estimation for the inverse Weibull; Salem et al. [95] treated the generalized inverted exponential; and Alotaibi et al. [96] investigated the Xgamma distribution. It is evident that there is a limited body of work addressing competing risks models within the framework of the APT-IHC scheme. In particular, no studies have examined dependent competing risks model, despite their significant practical relevance. Furthermore, the estimation of reliability and survival measures for additional commonly utilized lifetime models remains unexplored and necessitates further investigation.

Inference under the APT-IHC scheme combined with ALTs has received comparatively limited attention. Notable contributions include Ismail [97], who analyzed step-stress partially ALTs for the Weibull model and developed classical estimators for both model parameters and the acceleration factor; Alam et al. [98], who studied step-stress partially ALTs for the odd Lindley half-logistic distribution from a classical perspective; and Nassar and Elshahhat [99], who considered constant-stress partially ALTs for the inverse Weibull distribution using maximum likelihood, maximum product of spacings, and Bayesian approaches based on both the likelihood and spacings functions via MCMC procedure. In this regard, further investigation is warranted concerning constant-stress ALTs, step-stress ALTs, and progressive-stress ALTs. Additionally, incorporating the application of Bayesian estimation is recommended. This approach is particularly advantageous in scenarios where the number of observed failures is limited, potentially leading to inaccuracies in classical estimates.

4 AGPT-IIC Scheme

In the PT-IIC plan, a common practical situation is that the first r failures are unobserved, where $1 \leq r < n$ and $r < m$ (with n units on test and a target of m observed failures). This arises, for example, in post-warranty analysis when a few items fail before the warranty expires and are treated as unobserved, or

in clinical studies of time to cure when some patients die before enrollment and their failure times are unobserved. To accommodate such cases, the general PT-IIC scheme is employed: it explicitly allows the first r failure times to be missing and proceeds with progressive removals and likelihood construction based on the remaining $m - r$ observed failures, thereby preserving the PT-IIC structure while accounting for early, unrecorded failures, see for more detail Xiuyun and Zaizai [100].

Abu-Moussa et al. [15] introduced the AGPT-IIC scheme as an enhancement of the general PT-IIC plan, incorporating adaptability to optimize life-testing experiments. The scheme is outlined as follows: Place n identical units on test. The first r failures are unobserved, and the aim is to observe m failures with $r < m < n$. A progressive removal vector is fixed as $\mathbf{R} = (R_{r+1}, \ldots, R_m)$, with the condition $n = r + m + \sum_{i=r+1}^{m} R_i$. An ideal time threshold $T > 0$ is set in advance. At the $(r+1)$-st observed failure time $X_{r+1:m:n}$, withdraw R_{r+1} survivors; at $X_{r+2:m:n}$, withdraw R_{r+2}; and so on through $X_{m-1:m:n}$. The experiment terminates under one of two scenarios:

1. **Case I:** If $X_{m:m:n} < T$, the experiment stops at $X_{m:m:n}$ and the pre-specified removal pattern $\mathbf{R}$ is fully applied (the usual general PT-IIC case with r unobserved early failures).
2. **Case II:** If $X_{m:m:n} > T$, suspend scheduled removals after time T (i.e., no further withdrawals beyond T) and continue observing until the m-th failure occurs at $X_{m:m:n}$; then terminate the test and withdraw all remaining survivors.

The schematic representation of AGPT-IIC scheme is presented in Figure 3. The resulting failure times under this scheme are $X_{r+1:m:n}, X_{r+2:m:n} \ldots, X_{m:m:n}$, with the following progressive censoring plan

$$R_i = \begin{cases} R_i, & i = r+1, \ldots, d, \\ 0, & i = d+1, \ldots, m-1, \\ R_m^{\star} = n - m - \displaystyle\sum_{i=r+1}^{d} R_i \geq 0, & i = m, \end{cases}$$

where d denotes the number of failures observed before time T. Let $\mathbf{x}$ be an observed AGPT-IIC sample from a continuous model with PDF $f(x;\boldsymbol{\theta})$ and CDF $F(x;\boldsymbol{\theta})$. Then, ignoring the combinatorial constant, the LF is

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = [F(x_{r+1};\boldsymbol{\theta})]^r \prod_{i=r+1}^{m} f(x_i;\boldsymbol{\theta}) \prod_{i=r+1}^{d} [1 - F(x_i;\boldsymbol{\theta})]^{R_i} [1 - F(T;\boldsymbol{\theta})]^{R_m^{\star}}. \tag{10}$$

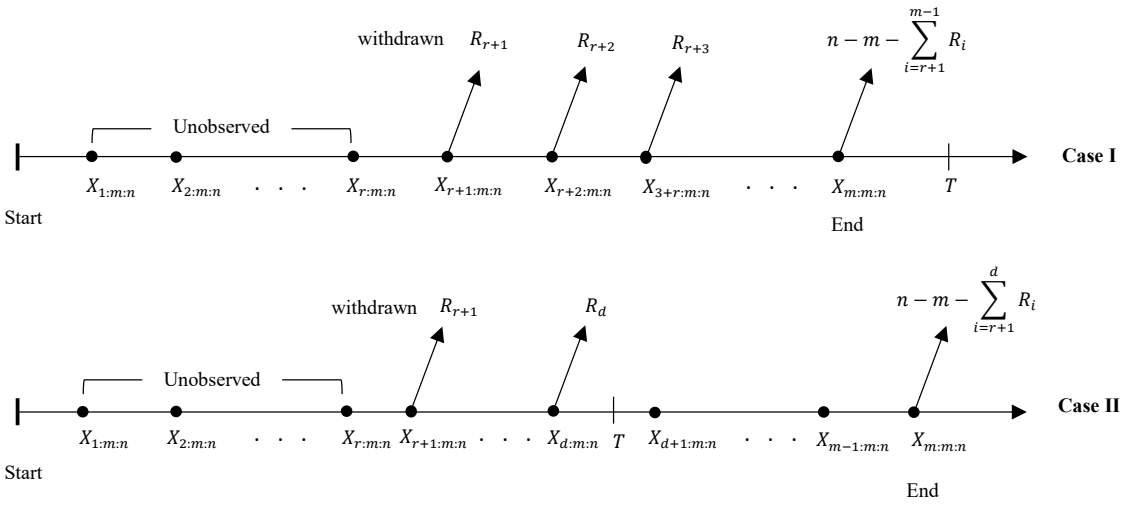

Figure 3: Schematic representation of AGPT-IIC scheme.

Abu-Moussa et al. [15] applied the AGPT-IIC plan to the Gompertz distribution, developing both classical estimators and Bayesian procedures under four loss functions. Gangopadhyay et al. [101] used the same AGPT-IIC framework to investigate reliability characteristics of the inverse Xgamma distribution.

Despite the practical appeal of the AGPT-IIC plan, the literature appears limited to the two studies noted above. This gap suggests substantial scope for future work, including developing inference for widely used lifetime models (e.g., Weibull, log-normal, Burr X-II), extending the framework to competing risks (independent and dependent), integrating with ALTs, and exploring design/optimality criteria and Bayesian methods for small-sample settings.

5 IAPT-IIHC Scheme

When the total test duration is not a binding constraint, the APT-IIHC scheme, by guaranteeing a pre-specified number of failures m, is highly efficient for statistical inference. However, for high reliability products the time required to accrue m failures can be prohibitively long, so the APT-IIHC scheme does not ensure a satisfactory overall test length. However, in numerous real-world scenarios, the duration of the experiment is a critical factor that must be taken into account. To address this constraint, Yan et al. [16] introduced the IAPT-IIHC scheme, which ensures the experiment concludes within a designated timeframe while also serving as a generalization of existing censoring schemes, including the APT-IIHC scheme. This section provides an in-depth overview of the scheme and its applications.

5.1 Design, likelihood, and spacings formulations

Consider a life test involving n units, with a predetermined number of failures $m < n$, dual time thresholds T_1 and T_2 (where $T_1 < T_2$), and a censoring plan $\mathbf{R} = (R_1, \ldots, R_m)$. The procedure is as follows: At the first failure time $X_{1:m:n}$, R_1 surviving units are randomly removed. At the second failure time $X_{2:m:n}$, R_2 surviving units are removed, and this process continues for subsequent failures. The experiment can terminate under three scenarios:

1. **Case I:** If $X_{m:m:n} < T_1$, the test stops at $X_{m:m:n}$, resulting in the PT-IIC scheme.
2. **Case II:** If $T_1 < X_{m:m:n} < T_2$, the test follows the APT-IIHC scheme. After observing d_1 failures before T_1, no further units are removed until the m-th failure, at which point all remaining units are withdrawn.
3. **Case III:** If $T_2 < X_{m:m:n}$, the test ends at T_2. Similar to Case II, no units are removed after getting d_1 failures, and at T_2, all surviving units are removed.

Let $\mathbf{x}$ represent an observed IAPT-IIHC sample from a continuous population with PDF $f(x;\boldsymbol{\theta})$ and CDF $F(x;\boldsymbol{\theta})$ under removal design $\mathbf{R}$. The joint LF can be formulated in a unified manner to encompass all three stopping scenarios as follows

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{D} f(x_i;\boldsymbol{\theta}) \prod_{i=1}^{J} [1 - F(x_i;\boldsymbol{\theta})]^{R_i} \left[1 - F(T^\star;\boldsymbol{\theta})\right]^{R^\star}, \tag{11}$$

where x_i is the realization of $X_{i:m:n}$ for simplicity of notation,

$$(D, J) = \begin{cases} (m, m-1), & \text{Case I,} \\ (m, d_1), & \text{Case II,} \\ (d_2, d_1), & \text{Case III,} \end{cases} \quad \text{and} \quad (T^\star, R^\star) = \begin{cases} \left(x_m,\ n - m - \sum_{i=1}^{m-1} R_i\right), & \text{Case I,} \\ \left(x_m,\ n - m - \sum_{i=1}^{d_1} R_i\right), & \text{Case I,} \\ \left(T_2,\ n - d_2 - \sum_{i=1}^{d_1} R_i\right), & \text{Case III,} \end{cases}$$

where d_1 and d_2 refer to the number of observed failures before the threshold T_1 and T_2, respectively. Figure 4 illustrates the three scenarios of the IAPT-IIHC scheme, showing the termination point for each case and the number of removals at the end. Yan et al. [16] investigated the estimation of the Burr-XII distribution, including certain reliability metrics, using the IAPT-IIHC scheme. Nassar and Elshahhat [102] explored classical estimation of the Weibull model through likelihood and maximum product of spacings methods, alongside Bayesian estimation. Employing the same notation, they defined the SF to be maximized under the IAPT-IIHC scheme as follows

$$\mathcal{P}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{D+1} [F(x_i;\boldsymbol{\theta}) - F(x_{i-1};\boldsymbol{\theta})] \prod_{i=1}^{J} [1 - F(x_i;\boldsymbol{\theta})]^{R_i} \left[1 - F(T^\star;\boldsymbol{\theta})\right]^{R^\star}. \tag{12}$$

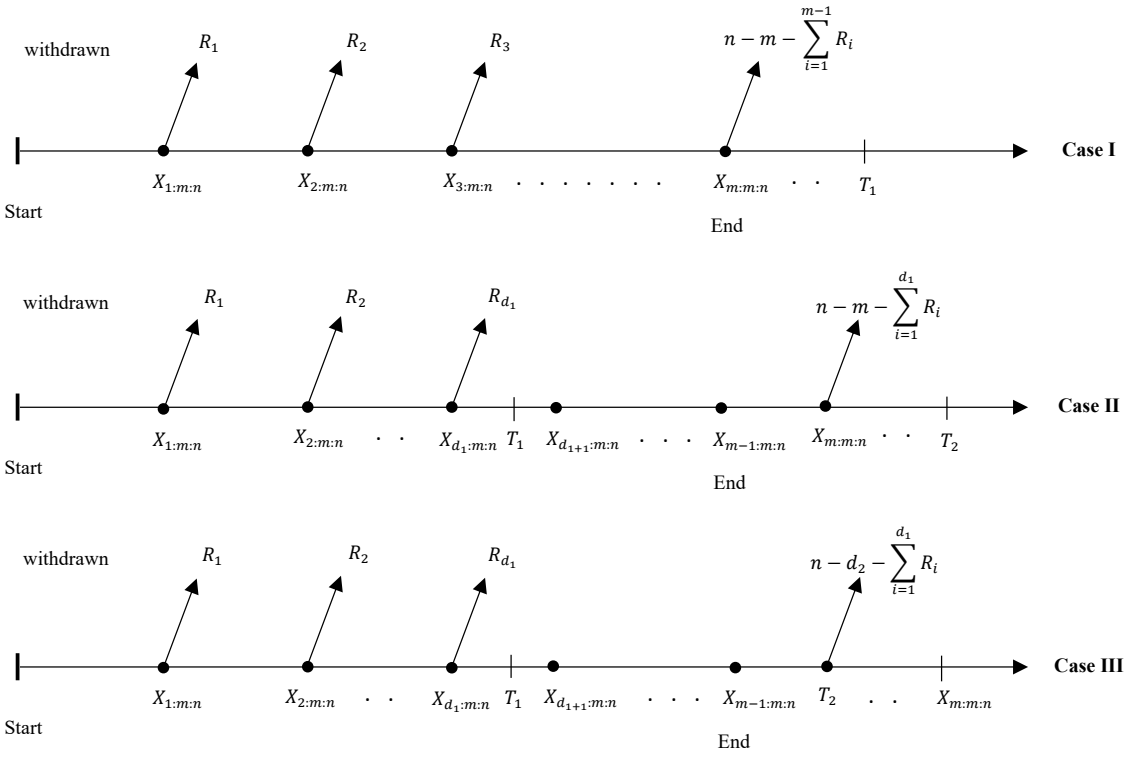

Figure 4: Schematic representation of IAPT-IIHC scheme.

5.2 Inference based on IAPT-IIHC scheme

This section reviews the body of research on statistical inference using the IAPT-IIHC scheme. It covers applications ranging from single-cause failure and competing risks to ALts.

Numerous studies have explored the application of the IAPT-IIHC scheme to various lifetime models, focusing on estimating unknown parameters, reliability/survival metrics, and determining optimal progressive censoring plans. Elbatal et al. [103] performed a reliability analysis for the alpha power exponential distribution. They derived various reliability measures and statistical properties for this model. Zhang and Yan [104] developed methods for estimating the parameters of the Chen distribution. They compared the performance of different estimation techniques using simulation studies. Dutta et al. [105] explored both Bayesian and Non-Bayesian inference for the logistic-exponential distribution. They derived estimators and constructed confidence intervals for the model's parameters. Elbatal et al. [106] performed Bayesian and E-Bayesian reliability analysis for the inverted Lindley distribution. They derived posterior summaries and reliability estimates under squared error loss function. El-Sherpieny et al. [107] conducted a statistical analysis of Nadarajah-Haghighi model. They focused on parameter estimation and model validation for the given dataset. Irfan et al. [108] developed statistical inference methods for the Kumaraswamy-G family of distributions. They also determined optimal plans for life-testing experiments based on this model. Swaroop et al. [109] estimated the stress-strength reliability for the generalized inverted exponential distribution. They derived the reliability estimator and evaluated its performance through simulations. Lone [110] estimated the parameters of the Burr Type III model. The methodology was demonstrated through a practical application to engineering data.

The IAPT-IIHC scheme has been widely adopted for statistical inference in competing risks models involving various lifetime distributions. Research in this area typically centers on estimating parameters and reliability characteristics from both classical and Bayesian perspectives. Illustrating this, Dutta and Kayal [111] performed inference on the exponential distribution and selected an optimal censoring strategy based on multiple optimality criteria. Within the previously established competing risks framework

involving two distinct causes of failure, the LF for the observed IAPT-IIHC sample can be formulated as follows

$$\begin{aligned}\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) &= \prod_{i=1}^{D}\left[f_1(x_i;\boldsymbol{\theta})\bar{F}_2(x_i;\boldsymbol{\theta})\right]^{\mathbb{I}(\phi_i=1)}\left[f_2(x_i;\boldsymbol{\theta})\bar{F}_1(x_i;\boldsymbol{\theta})\right]^{\mathbb{I}(\phi_i=2)}\\ &\times \prod_{i=1}^{J}\left[\bar{F}_1(x_i;\boldsymbol{\theta})\bar{F}_2(x_i;\boldsymbol{\theta})\right]^{R_i}\left[\bar{F}_1(T^{\star};\boldsymbol{\theta})\bar{F}_2(T^{\star};\boldsymbol{\theta})\right]^{R^{\star}}. \qquad (13)\end{aligned}$$

The total number of failures, D, is partitioned into D_1 and D_2, which denote the failures from the first and second causes, respectively. These values are defined by $D_1 = \sum_{i=1}^{D}\mathbb{I}(\phi_i = 1)$ and $D_2 = \sum_{i=1}^{D}\mathbb{I}(\phi_i = 2)$, and satisfy the condition $D_1 + D_2 = D$. Elshahhat and Nassar [112] conducted inference for Weibull lifetime models within a competing risks framework. They utilized both classical and Bayesian methods to estimate model parameters and reliability characteristics. Dey and Kayal [113] focused on the Chen lifetime distribution under a competing risks model. The researchers developed both classical and Bayesian estimation procedures for the unknown parameters. Alotaibi et al. [114] presented a reliability analysis for an independent Burr-X competing risks model. The study derived and compared reliability estimators using different statistical methods. See also the work of Dutta and Kayal [115].

The application of ALTs within the IAPT-IIHC framework has received limited attention. A notable investigation by Nassar et al. [116] focused on constant-stress ALTs for the Weibull distribution, employing both classical and Bayesian methods to estimate parameters and reliability function under normal operating conditions. To the best of our knowledge, other types of ALTs have not been explored under this censoring scheme, representing a significant area for future research.

6 APFFC Scheme

Wu and Kuş [117] proposed a life test plan called the progressive first-failure censoring scheme, which merges the concepts of first-failure and progressive censoring. The method begins with n independent groups of k items. Upon the first failure, the affected group and R_1 other randomly selected groups are removed. This process repeats for each subsequent failure, removing the group with the failure and a pre-specified number of other groups $(R_2, \ldots, R_m)$. The experiment concludes at the m-th failure $(m < n)$, where the final group and R_m others are withdrawn. Using this plan, Wu and Kuş [117] derived point and interval estimates for the parameters of the Weibull distribution. The progressive first-failure censoring scheme has recently garnered significant academic interest, with notable contributions from researchers such as Shi and Shi [118] and Kumar et al. [119], among others.

Fathi [18] recently introduced the APFFC Scheme, an extension of the standard progressive first-failure censoring methodology. This new plan provides experimenters with the flexibility to manage the total test duration and dynamically adjust censoring numbers in response to real-time conditions observed during the experiment. Consider a life test conducted on n independent groups, each comprising k items. The experimenter establishes a predefined optimal total test time, denoted by T, as well as the removal pattern $(R_1, \ldots, R_m)$. The procedure follows the APFFC, which operates as follows: upon observing the i-th failure time $X_{i:m:n}$ within any group, the remaining items from that group, along with R_i other randomly selected groups, are withdrawn from the experiment. The experiment terminates under one of two conditions:

1. **Case I:** If the m-th failure time $X_{m:m:n}$ occurs before the predefined time T, the experiment concludes at $X_{m:m:n}$. This scenario is identical to the conventional progressive first-failure censoring scheme.
2. **Case II:** If the experiment reaches time T without observing m failures, the experiment adapts. To ensure the experiment concludes with the target m failures without excessive delay, the censoring process is suspended. No further groups are withdrawn, and the test proceeds until the m-th failure is observed.

This adaptive strategy is designed to keep the total test duration close to the optimal time T while ensuring the desired number of failures is collected. Consequently, the expected total test time under

this adaptive scheme is shorter than that of its non-adaptive counterpart. The resulting failure times are $X_{1:m:n}, \ldots, X_{m:m:n}$ with a removal pattern $(R_1, \ldots, R_m)$. In the adaptive scenario where the experiment exceeds the time threshold T (i.e., $T < X_{m:m:n}$), no further groups are removed after time T. This situation is represented by setting $(R_{d+1}, \ldots, R_{m-1}) = 0$, where d denotes the number of failures observed prior to T. Let $\mathbf{x}$ be the observed APFFC sample selected from a continuous population with PDF $f(x;\boldsymbol{\theta})$ and CDF $F(x;\boldsymbol{\theta})$. Then, the joint LF can be formulated after ignoring the constant term as follows

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{m} f(x_i;\boldsymbol{\theta}) \prod_{i=1}^{d} [1 - F(x_i;\boldsymbol{\theta})]^{k(R_i+1)-1} \prod_{i=d+11}^{m-1} [1 - F(x_i;\boldsymbol{\theta})]^{k-1} [1 - F(x_m;\boldsymbol{\theta})]^{R_m^{\star}}, \quad (14)$$

where $R_m^{\star} = k\left(n - m + 1 - \sum_{i=1}^{d} R_i\right) - 1$. It is important to mention here that, once the experiment surpasses the predefined time threshold T, the removal of the R_i groups is halted. However, for each subsequent failure from $X_{d+1:m:n}$ to $X_{m-1:m:n}$, the group in which the failure occurs, containing its $(k-1)$ surviving items, is still withdrawn from the test. The two possible scenarios of the APFFC scheme are depicted in Figure 5, which highlights the termination point and the final number of removals for each case. Fathi [18] investigated statistical inference for the Weibull distribution using the APFFC scheme, employing both classical and Bayesian approaches. They compared the various estimators through a simulation study and an analysis of a real dataset. The APFFC scheme is a relatively new development in the field of life-testing and censoring methodologies.

To date, no published studies have applied this scheme in practical reliability or lifetime data analyses. This indicates that the APFFC approach remains largely unexplored and requires further investigation. Future research should focus on extending its theoretical properties, developing estimation procedures under various lifetime distributions, and assessing its performance through simulation studies and real-world applications. Moreover, integrating the APFFC plan with ALTs, competing risks, and Bayesian frameworks could provide deeper insights into its flexibility and practical advantages.

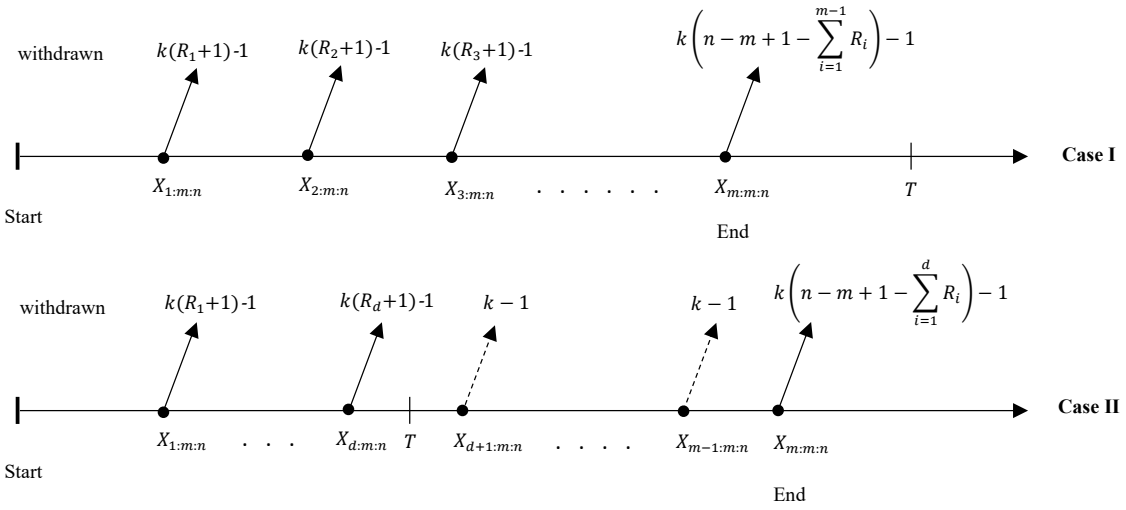

Figure 5: Schematic representation of APFFC scheme.

While the APFFC scheme improves upon the conventional progressive first-failure censoring scheme by enabling earlier test termination, its efficiency can be further increased. This is achieved by modifying the removal procedure after reaching the prefixed time T: instead of removing the group where a failure occurs, it is retained. This modification increases the number of active test units by $(m-d-1)(k-1)$ items after the pre-specified time T, compared to the APFFC scheme. By retaining more surviving items, this adaptation accelerates the test's conclusion after time T, terminating the experiment as soon as the pre-determined m failures are observed. In this scenario, there are two possible cases for test termination. The first case is identical to Case I of the APFFC scheme, as illustrated in Figure 5. The second case is a

modified version, as shown in Figure 6. In this case, the LF of the observed data can be written as follows

$$\mathcal{L}(\boldsymbol{\theta};\mathbf{x}) = \prod_{i=1}^{m} f(x_i;\boldsymbol{\theta}) \prod_{i=1}^{d} [1 - F(x_i;\boldsymbol{\theta})]^{k(R_i+1)-1} [1 - F(x_m;\boldsymbol{\theta})]^{R_m^{\star}}, \tag{15}$$

where $R_m^{\star} = nk - m - \sum_{i=1}^{d} [k(R_i + 1) - 1]$.

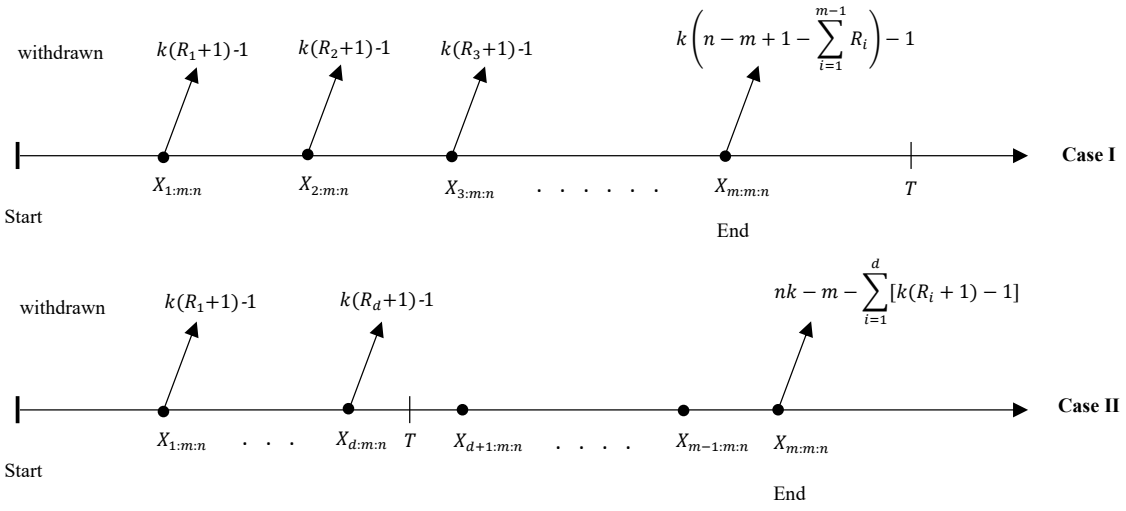

Figure 6: Schematic representation of modified APFFC scheme.

7 Conclusion

This review provided a comprehensive overview of adaptive progressive censoring schemes and their extensions, with a particular focus on their applications in reliability and survival analysis. The paper summarized the theoretical developments, estimation procedures, and inferential approaches that had been proposed for a variety of lifetime models under different censoring designs. It discussed how researchers applied both classical and Bayesian frameworks to estimate model parameters, reliability functions, and hazard rate measures, and how the choice of censoring strategy influenced the efficiency of statistical inference. The review also highlighted how adaptive designs had been successfully integrated with complex modeling frameworks such as competing risks analysis, accelerated life testing, and random removal mechanisms, thereby broadening their practical scope in engineering and biomedical studies. Furthermore, several challenges and open issues were identified, including the need to explore dependent risk structures, optimal censoring designs, and more computationally efficient estimation algorithms. Overall, this study concluded that adaptive progressive censoring strategies had significantly advanced the field of reliability and survival data analysis. Nevertheless, there remained considerable potential for further research aimed at improving model flexibility, optimizing design efficiency, and extending current methodologies to real-world testing environments involving multiple sources of uncertainty. One promising direction for future research is the comprehensive comparison of the various adaptive progressive censoring plans. To date, no study has fully, or even partially, investigated the comparative performance of adaptive progressive schemes. Although such an undertaking would require substantial analytical and computational effort, it remains feasible to begin this comparison incrementally by focusing on selected censoring plans as a foundation for broader future work.

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Data Availability

No new data were generated or analyzed in this study.

Conflicts of Interest:

The author declares no conflicts of interest regarding this study.

Author Contributions

The author is solely responsible for all components of this work.

AI Usage Statement:

No AI-based tools were used in the generation, writing, revising, analysis, or interpretation of the research data and findings presented in this article.

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