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A Comprehensive Survey of Weibull-Based Families

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Abstract

The article presents a comprehensive survey of families based on the Weibull distribution, which are fundamental tools for data modeling across various fields. The study systematically collect and organize Weibull-based families according to the number of parameters. The review is structured in some sections, where each presents a group of families that share the same parameter dimensionality. This work serves as both a reference framework for practitioners selecting appropriate distributions and a foundation for researchers developing new Weibull-family extensions.

1 Introduction

The Weibull distribution (Weibull [1]) has established itself as one of the most fundamental and versatile continuous distributions in statistical literature. It has of the most widely used models for lifetime data (Murthy et al. [2] and Rinne [3]). Its primary limitation lies in its inability to capture nonmonotone hazard rates, particularly the commonly observed bathtub-shape. This limitation has motivated the development of various generalizations of the Weibull model.

Its initial application in modeling the strength of materials has expanded considerably over decades, making it a cornerstone for modeling positive continuous data in several fields. Its flexibility, characterized by its scale and shape parameters, enables it to describe some failure patterns and aging processes, which accounts for its widespread adoption in numerous scientific disciplines.

Despite its good utility, the conventional two-parameter Weibull distribution exhibits limitations in capturing complex real-world phenomena commonly observed in engineering systems and biological studies. This recognition prompted the development of numerous Weibull extensions. Early modifications began with the introduction of the Weibull distribution by Weibull [1], followed by more sophisticated extensions including the exponentiated Weibull by Mudholkar and Srivastava [4] and Mudholkar et al. [5], which demonstrated enhanced flexibility in the modeling of hazard rates. More information on this distribution can be explored in Nadarajah et al. [6].

Recent decades have witnessed an exponential growth in Weibull-related distributions, with significant contributions such as additive Weibull by Xie and Lai. [7], modified Weibull by Sarhan and Zaindin [8], and various compound models. In the last 50 years, numerous Weibull generalizations have been investigated to model complex lifetime data that cannot be adequately captured by the standard Weibull model. See Carta et al. [9].

The proliferation of Weibull-based distributions, while enriching the statistical toolbox, has created a pressing need for systematic classification and comparative analysis. Several review efforts have been

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undertaken, including the comprehensive survey in Pham and Lai [10] on generalizations and applications. However, the rapid development of new families and methodologies necessitates updated syntheses that incorporate recent theoretical advances and emerging application domains.

Given the fundamental importance of the Weibull distribution and its extensive applications in reliability, survival analysis, engineering, and other scientific fields, numerous authors have proposed diverse families of continuous distributions constructed as extensions or generalizations of the Weibull model. In the article, we provide an extensive survey of Weibull-derived families, serving as a modest effort by the authors to document and consolidate this broad and rapidly expanding body of work. By presenting an organized overview of all known Weibull-derived distributions, we hope to support researchers in identifying existing models, avoiding redundant proposals, and facilitating further theoretical and applied developments in this area.

These Weibull-based families have been collected and organized according to the number of parameters that characterize each distribution. Consequently, the review is structured into several sections, with each section presenting a group of families that share the same dimensionality—whether the model is one-parameter, two-, three-, four- or five-parameter.

This research addresses this gap by providing a comprehensive, up-to-date survey of Weibull-derived families. The survey aims to serve as both a reference framework for practitioners and a foundation for future research, while identifying promising avenues for further methodological development and practical innovation in Weibull-based statistical modeling.

1.1 Transformed-Transformer (T-X) Generator

A useful technique for developing flexible distributions is the T-X method, which was introduced by Alzaatreh et al. [11]. It is also referred to as the T-X family, which is a generalized approach to modify continuous distributions. It was designed in such a way that any continuous distribution can serve as a generator. The cumulative distribution function (CDF) of the T-X family is

$$F(x) = \int_a^{W[G(x)]} r(t)dt, \tag{1}$$

where $r(t)$ is the probability density function (PDF) of $T \in [a, b]$ (for $a < b$) and $W[G(x)]$ is a function of the CDF of X. Some weak conditions of this function hold (see Alzaatreh et al. [11]).

Researchers have introduced several new classes of distributions from (1) in recent years. This approach has proven particularly effective in generating extended forms of well-known distributions. Table 1 highlights some of the most commonly employed transformation functions and parametrizations adopted to construct various families of Weibull-related models within the T-X framework. The transformation domains are equal to the domain of $G(x)$.

In the next sections, several relevant generalized Weibull classes are described in a very summarized way.

2 Weibull-Based Generators

This section provides a summary of the literature on various families based on Weibull distribution. The support of all families is equal to the support of the parent $G(x)$. So, we have omitted the definition of the supports. We present only the cumulative distributions because calculating the corresponding densities involves only differentiation. All parameters in the classes below are positive unless otherwise noted. We do not mention the shapes of the cumulative and density functions since these are available in the cited papers.

The next sections are organized based on the number of parameters in each family—whether the model includes one, two, three, four, or even five parameters.

Table 1: Some transformation functions within the T-X framework.

No.	$W(\cdot)$
1	$G(x)$
2	$\frac{G(x)}{\bar{G}(x)}$
3	$\frac{G(x)^{\alpha}}{1-G(x)^{\alpha}}$
4	$\left[\frac{\bar{G}(x)}{G(x)}\right]^{\alpha}$
5	$\frac{[1-\bar{G}^2(x)]^b}{1-[1-\bar{G}^2(x)]^b}$
6	$\frac{G(x)}{\{\bar{G}(x)[1+G(x)]\}/2}$
7	$-\log[G(x)]$
8	$-\log[1-G(x)^c]$
9	$-\log\left[1-\bar{G}(x)^{\alpha}\right]$
10	$\frac{-\log[1-G(x)]}{\bar{G}(x)}$
11	$1-\exp\left[-\theta G(x)\right]$
12	$1-\exp\left\{-\alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\lambda}\right\}$
13	$-\log\left[1-\exp\left(1-\left\{1+\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\beta}\right)\right]$
14	$G\left(x\right)\left[2-G\left(x\right)\right]/\bar{G}\left(x\right)$

3 One-Parameter Families

3.1 Type-I Weibull-G (TIW-G) Family

The TIW-G by Ahmad et al. [12] provides a flexible extension of Weibull. Two sub-models were reported, and the parameters were estimated using ordinary least squares, percentile estimation, and maximum likelihood. Several mathematical properties were obtained, and the utility of the class was proved through two real-world applications. Its CDF follows

$$F(x)=1-\exp\left\{-\left[\frac{-\log\bar{G}\left(x\right)}{\bar{G}\left(x\right)}\right]^{\alpha}\right\},$$

where $\bar{G}(x)=1-G(x)$ from now on.

4 Two-Parameter Families

4.1 Marshall-Olkin Extended Weibull-G (MOEW-G) Family

The CDF of MOEW-G by Santos-Neto et al. [13] is

$$F\left(x\right)=\frac{1-\exp\left[-\alpha G\left(x\right)\right]}{1-\bar{\delta}\exp\left[-\alpha G\left(x\right)\right]},$$

where $\bar{\delta}=1-\delta$.

The MOEW by Ghitany et al. [14] follows when $G(x)=x^{\gamma}$. The MOEW-Weibull and MOEW-Gompertz are two special cases. By comparing some of its sub-models with well-known lifetime distributions for fitting two real datasets, the special models provide better fits.

4.2 Truncated Weibull-G (TW-G) Family

The CDF of TW-G by Najarzadegan et al. [15] is

$$F(x) = \frac{1 - \exp\left[-\alpha G(x)^{\beta}\right]}{1 - \exp(-\alpha)}.$$

The TW-G family includes important special models such as TW-Weibull, TW-Logistic, TW-Log-Logistic, TW-Burr XII, and TW-Normal distributions. This class has (in general) better performance than the Beta-G (B-G) class.

4.3 Transmuted Weibull-G (TrW-G) Family

The CDF of TrW-G by Alizadeh et al. [16] is

$$F(x) = \left(1 - \exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}\right)\left(1 + \lambda \exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}\right), \quad |\lambda| \leq 1.$$

The authors provided structural properties and discussed the TrW-Weibull and TrW-Lindley distributions. The estimation was done by maximum likelihood. Its usefulness and effectiveness were proved through applications to two real datasets.

4.4 Extended Odd Weibull-G (ExOW-G) Family

The CDF of ExOW-G by Alizadeh et al. [17] is

$$F(x) = 1 - \left\{1 + \beta\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\frac{-1}{\beta}}.$$

Mathematical properties, special cases (ExOW-Normal and ExOW-Weibull) and parameter estimation were provided. Analysis of two real datasets showed that these two models provide better fits than other classes.

4.5 Generalized Odd Weibull-G (GOWG-G) Family

The CDF of GOWG-G by Mustafa et al. [18] is

$$F(x) = 1 - \exp\left\{-\left[\frac{G(x)^{\alpha}}{1 - G(x)^{\alpha}}\right]^{\beta}\right\}.$$

The GOWG-Normal and GOWG-W distributions were reported. The estimation of parameters, its practicality and an application to two real datasets were addressed.

4.6 Weibull Marshall–Olkin-G (WMO-G) Family

The CDF of WMO-G by Korkmaz et al. [19] is

$$F(x) = 1 - \exp\left(-\left\{-\log\left[\frac{\alpha\bar{G}(x)}{1 - \bar{\alpha}\bar{G}(x)}\right]\right\}^{\beta}\right),$$

where $\bar{\alpha} = 1 - \alpha$.

Its flexibility was proved by means of three real-world datasets. Two subfamilies: W-X (Alzaatreh et al. [11], Cordeiro et al. [20]) for $\alpha = 1$ and MO-G by Marshall and Olkin [21] if $\beta = 1$. Some properties were addressed for WMO-Fréchet and WMO-exponential.

4.7 Odd Flexible Weibull-G (OFW-G) Family

The CDF of OFW-G by El-Morshedy and Eliwa [22] is

$$F(x) = 1 - \exp\left\{-\exp\left[\alpha\frac{G(x)}{\bar{G}(x)} - \beta\frac{\bar{G}(x)}{G(x)}\right]\right\}.$$

Its usefulness was verified for three real data sets. The OFW-Fréchet and OFW-exponential were discussed. The biases and mean squared errors (MSEs) of the maximum likelihood estimates (MLEs) were reported.

4.8 Modified Odd Weibull-G (MOW-G) Family

The CDF of MOW-G by Chesneau and El Achi [23] is

$$F(x) = 1 - \exp\left(-\lambda\left\{\frac{G(x)}{\frac{\bar{G}(x)[1+G(x)]}{2}}\right\}^{\theta}\right).$$

The MOW-normal and MOW-gamma exhibit flexible density functions capable of producing symmetric, unimodal, and bimodal shapes. A Monte Carlo simulation verified the precision of MLEs. The MOW-gamma distribution was better among those tested.

4.9 Exponentiated Truncated Inverse Weibull-G (ETIW-G) Family

The CDF of ETIW-G by Almarashi et al. [24] is

$$F(x) = \left\{1 - \exp\left[1 - \bar{G}(x)^{-b}\right]\right\}^{a}.$$

The ETIW-inverse Weibull outperformed six competing models for three real data sets.

4.10 Weibull-G Poisson (W-GP) Family

The CDF of W-GP by Yousof et al. [25] is

$$F(x) = \frac{1}{[\exp(\theta) - 1]}\left[\exp\left(\theta - \theta\exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}\right) - 1\right].$$

Its application to three real datasets provided better fits compared to models obtained from known families. The W-Gamma-P, W-exponentiated exponential-P, and W-log-logistic-P were addressed. The precision of the MLEs was explored.

4.11 Odd Log-Logistic Weibull-G (OLLW-G) Family

The CDF of OLLW-G was identified and its statistical characteristics were examined by Rasekhi et al. [26]. Its CDF is

$$F(x) = \frac{\left(1 - \exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}\right)^{\beta}}{\left(1 - \exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}\right)^{\beta} + \exp\left\{-\beta\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}}.$$

Two models based on Weibull and normal baselines were discussed. The results indicate that the models outperform traditional methods for censored lifetime data and financial risks.

4.12 Harmonic Mixture Weibull-G (HMW-G) Family

The CDF of HMW-G by Zamanah et al. [27] is

$$F(x) = 1 - \frac{\exp\left\{-\alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta}\right\}}{1-\theta\left(1-\exp\left\{-(\alpha-1)\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta}\right\}\right)}, \quad 0<\theta<1.$$

Five distributions (HMW-BIII, HMW-L, HMW-W, HMW-F, HMW-N) were analyzed, thus showing appealing shapes suitable for medical research. The precision of the MLEs is investigated under Monte-Carlo simulations. It was developed a location-scale regression model and applied to survival times of hypertensive patients.

4.13 Truncated Cauchy Power Weibull-G (TCPW-G) Family

The CDF of TCPW-G by Alotaibi et al. [28] is

$$F(x) = \frac{4}{\pi}\arctan\left(1-\exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\lambda}\right\}\right)^{\theta}.$$

Type-II censored samples were adopted for maximum likelihood and Bayesian estimations, and properties of Bayes estimators were analyzed under different loss functions. A simulation study showed that Bayesian estimates often outperformed others. A specific case based on the Lomax distribution is applied to carbon fiber and Covid-19 data for comparing several models.

4.14 Kavya–Manoharan Weibull-G (KMW-G) Family

The CDF of KMW-G family by Elbatal et al. [29] is

$$F(x) = \frac{\mathrm{e}}{\mathrm{e}-1}\left\{1-\exp\left[-\left(1-\exp\left\{-\alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta}\right\}\right)\right]\right\}.$$

Some mathematical properties of this class were obtained. Bayesian (for symmetric loss functions) and maximum likelihood methods were reported for progressive type-II censoring (PTIIC). Two actuarial datasets validated the KMW-Lomax model against seven established models. For the PTIIC-based KMW-exponential model, Bayesian and maximum likelihood techniques were developed to find asymptotic confidence intervals.

4.15 Type-II Weibull-G (TIIW-G) Family

The TIIW-G family and some structural properties of four special models were developed by Afify et al. [30]. Its CDF has the form

$$F(x) = 1 - \exp\left(-\alpha\left\{\frac{G(x)\,[2-G(x)]}{\bar{G}(x)}\right\}^{\beta}\right).$$

The exponential-G (Ex-G) and Rayleigh-G (R-G) come when $\beta = 1$ and $\beta = 2$, respectively (Jamal et al. [31]).

Estimation was conducted using ten different methods, covering both classical and Bayesian frameworks under various loss functions. Simulations revealed that Bayesian estimators often provided superior performance compared to other methods. The usefulness of the W-Ex distribution was proved using three datasets. This distribution exhibited superior goodness-of-fit and greater modeling flexibility compared to several exponential-type models.

4.16 Odd-Generalized Rayleigh-Reciprocal Weibull-G (OGR-RW-G) Family

The CDF of OGRRW-G by Musekwa et al. [32] is

$$F(x) = 1 - \exp\left\{-\left[\frac{e^{-2\alpha\nabla(x)^{-\beta}}}{\left(1 - e^{-2\alpha\nabla(x)^{-\beta}}\right)^2}\right]\right\},$$

where $\nabla(x) = G(x)/\bar{G}(x)$.

Some of its properties were studied. The OGR-RW-BXII model significantly showed the utility of the family.

4.17 Weibull Sine Generalized-G (WSG-G) Family

The cdf of WSG-G by Obulezi et al. [33] is

$$F(x) = 1 - \exp\left(-\left\{\frac{\sin\left[\frac{4}{\pi}G(x)\right]}{\rho}\right\}^{v}\right).$$

The WSG-E distribution allows for heavier tails and better accommodation of outliers, and it is robust for skewed data.

4.18 Odd Weibull-G (OW-G) Family

The CDF of OW-G by Ahmed and Khaleel [34] is

$$F(x) = 1 - \exp\left(-\alpha\left\{-\frac{G(x)}{\bar{G}(x)}\log\left[\frac{1}{\bar{G}(x)}\right]\right\}^{\beta}\right).$$

The usefulness of the OWIW and OWK models has been proven in two applications.

4.19 Topp–Leone Odd Weibull Flexible-G (TLOWF-G) Family

The TLOWF-G was studied by Chipepa et al. [35], exploring its mathematical properties. Its CDF is

$$F(x) = \left(1 - \exp\left\{-2\left[\bar{G}(x)^{-G(x)} - 1\right]^{\alpha}\right\}\right)^{b}.$$

It gives a better fit than competing models for two real datasets. A simulation study evaluated the accuracy of the MLEs.

4.20 Cosine Weibull-G (CW-G) Family

The CDF of CW-G by Zamanah and Nasiru [36] is

$$F(x) = 1 - \cos\left[\frac{\pi}{2}\left(1 - \exp\left\{-\gamma\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta}\right\}\right)\right].$$

Five distributions (CW-BIII, CW-L, CW-W, CW-F, CW-N) were analyzed, thus showing appealing shapes suitable for medical research. The accuracy of the RMSE and other criteria was investigated under simulations. The findings for two medical datasets showed that the CWW and CWL distributions achieved better fits compared to several competing models.

5 Three-Parameter Families

5.1 Beta Extended Weibull-G (BEW-G) Family

The CDF of BEW-G by Cordeiro et al. [37] is

$$F(x) = I_{1-\exp[-\theta G(x)]}(a, b),$$

where $I_p(a, b)$ is the incomplete beta function ratio. This class offers great flexibility.

5.2 Exponentiated Weibull-G (EW-G) Family

The EW-G by Alzaghal et al. [38] has three additional parameters to achieve greater tractability. Its CDF is

$$F(x) = 1 - \exp\left(-\left\{-\frac{\log[1 - G(x)^c]}{\gamma}\right\}^{\alpha}\right).$$

Its flexibility was evaluated for three real datasets by comparing it with other distributions. The findings showed that the EW-G class effectively models both left-skewed and right-skewed datasets. It has a wide range of models for fixed parameters; see Table 2.

Table 2: Some sub-models of the EW-G class

c	α	γ	Family	Author
–	1	1	E-G	Gupta et al. [39]
–	–	$\frac{1}{\gamma}$	KW-G	Cordeiro and De Castro [40]
1	–	–	W-G	Alzaatreh and Ghosh [41]

5.3 Generalized Extended Weibull Power Series (GEWPS) Family

The CDF of GEWPS by Alkarni [42] is

$$F(x) = \frac{C\left\{\theta\left[1 - e^{-\alpha G(x)}\right]^{\delta}\right\}}{C(\theta)},$$

where $C(\theta) = \sum_{n=0}^{\infty} a_n \theta^n$ and a_n is a function of n.

The GEWPS includes the extended Weibull and generalized exponential power series classes. It has broad applications in business, environmental studies, biomedical research, demography, industrial and reliability, and many other fields. Its mathematical properties have been extensively analyzed, and parameter estimation is carried out using the expectation-maximization algorithm combined with the maximum likelihood method. To illustrate its flexibility and effectiveness, several of its sub-models were applied to real datasets, demonstrating better performance compared to existing models.

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5.4 Gamma Weibull-G (GaW-G) Family

The CDF of GaW-G by Oluyede et al. [43] is

$$F(x) = \gamma_1\left(\delta, \alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta}\right),$$

where $\gamma_1(\delta, x)$ is the incomplete gamma function ratio.

For $\delta = 1$, GaW-G becomes W-G by Bourguignon et al. [44]. It leads to GaE-G and E-G by Cordeiro et al. [45] when $\beta = 1$ and $\delta = \beta = 1$, respectively. The Ga-Rayleigh-G and Rayleigh-G correspond to $\beta = 2$ and $\beta = 2, \delta = 1$, respectively. The GaW-uniform and GaW-W models were applied to real data.

5.5 Extended Weibull-G (EW-G) Family

The CDF of EW-G by Korkmaz [46] is

$$F(x) = 1 - \exp\left\{-\alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta} e^{-\lambda\left[\frac{\bar{G}(x)}{G(x)}\right]}\right\}.$$

Its utility was shown through lifetime data applications. The EW-Uniform, EW-W and EW-normal were discussed. The EW-G becomes the W-G class when $\lambda = 0$, which is the extended exponential-G if $\beta = 1$.

5.6 Weibull Generalized-G (WG-G) Family

The CDF of WG-G by Oluyede et al. [47] is

$$F(x) = \exp\left(-\lambda\left\{-\log\left[1 - \bar{G}(x)^{\alpha}\right]\right\}^{\beta}\right).$$

It includes sub-models such as WG-Normal and WG-Weibull. Its density showed different shapes such as symmetric, unimodal, left-right skewed, and flexible hazard rates.

This family includes well-known families as special cases (Table 3).

Table 3: Some sub-models of the NWG-G family

α	β	Family	Author
1	–	W-G	Tahir et al. [48]
–	1	EG-G	–
–	2	RG-G	–
1	1	E-G	–
1	2	R-G	–

5.7 Odd Inverse Power Generalized Weibull-G (OIPGW-G) Family

The CDF of OIPGW-G by Al-Moisheer et al. [49] is

$$F(x) = \exp\left(1 - \left\{1 + \theta\left[\frac{G(x)}{\bar{G}(x)}\right]^{-\alpha}\right\}^{\lambda}\right).$$

This family includes special cases: OIPGW-Uniform, OIPGW-E, OIPGW-W, and OIPGW-Rayleigh. Some subclasses are listed in Table 4.

Table 4: Special models of the OIPGW-G family

λ	θ	α	Family	Author
1	–	–	OFr-G	UlHaq and Elgarhy [50]
–	–	1	OINH-G	–
1	–	1	OIE-G	–
1	–	2	OIR-G	–

5.8 Exponentiated Odd Weibull Topp–Leone-G (EOWTL-G) Family

The CDF of EOWTL-G by Chamunorwa et al. [51] is

$$F(x) = \left[1 - \exp\left(-\left\{\frac{\left[1 - \bar{G}^2(x)\right]^b}{1 - \left[1 - \bar{G}^2(x)\right]^b}\right\}^{\beta}\right)\right]^{\alpha}.$$

The special OWTL-G class follows when $\alpha = 1$ (Chipepa et al. [52]). The EOWTL-log-logistic and EOWTL-Kumaraswamy were studied. Two real datasets demonstrated their effectiveness, with the first one significantly outperforming other sub-models.

5.9 Odd Weibull Topp–Leone-G Power Series (OWTLGPS) Family

The CDF of OWTLGPS by Oluyede et al. [53] is

$$F(x) = 1 - \frac{1}{C(\theta)} C\left\{\theta\left[\exp\left(-\left\{\frac{\left[1-\bar{G}^2(x)\right]^{\gamma}}{1-\left[1-\bar{G}^2(x)\right]^{\gamma}}\right\}^{\alpha}\right)\right]\right\},$$

where $C(\cdot)$ was defined before.

Several mathematical properties of this class were obtained. Two real datasets showed that it outperforms several non-nested models.

5.10 Gamma Odd Generalized Weibull-G (GaOPGW-G) Family

The CDF of GaOPGW-G by Gabanakgosi et al. [54] is

$$F(x) = 1 - \gamma_1\left(\delta, -\log\left[1-\exp\left(1-\left\{1+\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\beta}\right)\right]\right).$$

The GaOPGW-BurrXII and GaOPGW-W are special cases of the current family. Their flexibility is demonstrated through real data. Some subclasses are listed in Table 5.

Table 5: Subclasses of the GaOPGW-G family

δ	β	α	Family	Author
1	1	–	One-parameter W-G	Bourguignon et al. [44]
1	–	–	OPGW-G	Oluyede et al. [55]
1	–	1	ONH-G	–
–	1	–	GaOE-G	–
–	–	1	GaONH-G	–
–	1	2	GaOR-G	–
1	1	2	OR-G	–
1	1	1	OE-G	–

5.11 Marshall–Olkin Odd Generalized Weibull-G (MOOGW-G) Family

The CDF of MOOGW-G by Chipepa et al. [56] is

$$F(x) = \frac{1-\exp\left(1-\left\{1+\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\beta}\right)}{1-\bar{\delta}\exp\left(1-\left\{1+\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\beta}\right)}.$$

It is suitable for fitting real data in various fields. Some subfamilies are reported in Table 6. The MOOGW-log-logistic model was applied to COVID-19 mortality rates, which outperforms some other models.

Table 6: Special cases of the MOOGW-G family

δ	α	β	Family	Author
1	–	1	One-parameter W-G	Bourguignon et al. [44]
1	1	1	OE-G	Bourguignon et al. [44]
1	2	1	OR-G	Bourguignon et al. [44]
1	1	–	ONH-G	Nascimento et al. [57]
1	–	–	OGW-G	Oluyede et al. [55]
–	–	1	MOW-G	Afify et al. [58]
–	1	–	MOONH-G	–
–	1	1	MOOE-G	–
–	2	1	MOOR-G	–

5.12 Exponentiated Weibull-G (EW-G) Family

The CDF of EW-G by Cordeiro et al. [59] is

$$F(x) = \left[1 - \exp\left(-\lambda^{-p}\left\{-\ln\left[1 - G(x)\right]\right\}^{p}\right)\right]^{\alpha}.$$

The utility of the current family was demonstrated with two applications to real COVID-19 datasets.

5.13 Modified Alpha Power Weibull-G (MAPW-G) Family

The CDF of MAPW-G by Emam and Tashkandy [60] is

$$F(x) = \frac{\alpha^{1-\exp\left\{-\left[\beta G(x)\right]^{\theta}\right\}} - 1}{\alpha^{1-\exp\left(-\beta^{\theta}\right)} - 1}, \quad \text{if } \alpha \neq 1.$$

Some structural properties of the five-parameter MAPW–Weibull were determined. The Bayesian estimators outperformed MLEs in a simulation study. This model was applied to predict patterns in a carbon dioxide emissions dataset. The Kolmogorov-Smirnov test indicated that it gives a better fit than other models.

5.14 Type I Heavy-Tailed Odd Power Generalized Weibull-G (TIHTOPGW-G) Family

The CDF of TIHTOPGW-G by Moakofi and Oluyede [61] is

$$F(x) = 1 - \left\{\frac{\exp\left(1 - \left\{1 + \left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\beta}\right)}{1 - \bar{\theta}\left[1 - \exp\left(1 - \left\{1 + \left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}^{\beta}\right)\right]}\right\}^{\theta},$$

where $\bar{\theta} = 1 - \theta$.

The TIHTOPGW-G class has some special cases listed in Table 7. Statistical properties were presented by including actuarial measures. The MLEs were determined and evaluated evaluated through. The family applied to four real datasets showed improved performance over various heavy-tailed distributions.

Table 7: Special cases of the TIHT-OPG-G family

θ	α	β	Family	Author
1	–	1	One-parameter W-G	Bourguignon et al. [44]
1	1	–	ONH-G	Nascimento et al. [57]
1	–	–	OPGW-G	Oluyede et al. [55]
–	–	1	TI-HT-W-G	–
–	1	–	TI-HT-ONH-G	–
1	1	1	OE-G	–
1	2	1	OR-G	–

5.15 Extended Odd Inverse Weibull-G (EOIW-G) Family

The CDF of EOIW-G by Abdelall et al. [62] is

$$F(x) = \exp\left(-\left\{\theta^{\alpha}\left[\frac{G(x)^{\beta}}{1-G(x)^{\beta}}\right]^{-\alpha}\right\}\right).$$

The parameters were estimated using Bayesian and classical methods and simulations were reported. The Bayesian estimates provide a better behavior than MLEs. An evaluation of two engineering datasets showed that the proposed model outperformed six existing models. This family also includes subclasses in Table 8.

Table 8: Some special cases of the EOIW-G family

θ	α	β	Family	Author
–	–	1	IW-G	Hassan and Nassr [63]
1	–	1	OF-G	UlHaq and Elgarhy [50]
1	–	–	EOF-G	Nasiru [64]
–	1	–	EOIE-G	–
–	2	–	EOIR-G	–

5.16 Marshall–Olkin Weibull-Exponentiated Half Logistic-G (MOWEHL-G) Family

The CDF of MOWEHL-G by Moakof et al. [65] is

$$F(x) = 1 - \frac{\delta \exp\left[-\left(-\log\left\{1-\left[\frac{G(x)}{1+\bar{G}(x)}\right]^{\alpha}\right\}\right)^{\beta}\right]}{1-\bar{\delta}\exp\left[-\left(-\log\left\{1-\left[\frac{G(x)}{1+\bar{G}(x)}\right]^{\alpha}\right\}\right)^{\beta}\right]}.$$

Three baselines were considered: logistic, Weibull, and standard half logistic. Some mathematical properties were derived, and six classical estimation methods were used to estimate the parameters, assessed through Monte Carlo simulations. Its adequacy was evaluated using three datasets.

5.17 Alpha Power Weibull-G (APW-G) Family

The CDF of APW-G by Baharith and Aljuhani [66] is

$$F(x) = \frac{1}{\alpha-1}\left[\alpha^{\left(1-\exp\left\{-\left[\frac{-\log\bar{G}(x)}{\beta}\right]^{c}\right\}\right)}-1\right], \text{ if } \alpha > 0, \alpha \neq 1$$

This family combines two techniques: transformed-transformer and alpha power transformation, allowing for vast flexibility. Its usefulness was shown by means of three datasets.

5.18 Inverse Lomax Weibull-G (ILW-G) Family

The CDF of ILW-G by Falgore et al. [67] is

$$F(x) = \left[1 + \lambda\left(\frac{\exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}}{1 - \exp\left\{-\left[\frac{G(x)}{\bar{G}(x)}\right]^{\alpha}\right\}}\right)\right]^{-\gamma}.$$

Some structural properties of the ILWW distribution were examined. Simulations were carried out to obtain the biases and RMSEs of the MLEs. In addition, a survival regression analysis was presented. The ILWW and log-ILWW models are very adequate for leukemia data.

5.19 Discrete Weibull-G (DW-G) Family

The CDF of DW-G family by Balubaid et al. [68] is

$$F\left(x\right) = 1 - \exp\left(-\left\{\frac{-\log\left[1 - G\left(x+1,\delta\right)\right]}{\beta}\right\}^{c}\right).$$

The DWE distribution was taken as a special case. Mathematical properties and simulations were reported. Its usefulness was proved by means of three real datasets.

6 Four and Five-Parameter Families

6.1 Kumaraswamy Weibull-G (KwW-G) Family

The CDF of KwW-G family by Hassan and Elgarhy [69] is

$$F(x) = 1 - \left(1 - \left[1 - \exp\left\{-\alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\beta}\right\}\right]^{a}\right)^{b}.$$

The W-G class (Bourguignon et al. [44]) follows when $a = b = 1$, while the EW-G class corresponds to $b = 1$. It was considered for baseline: uniform, BurrXII, Weibull, and quasi-Lindley. This family outperformed other established lifetime models. Table 9 reports some of its special cases

Table 9: Sub-models of the KwW-G class

α	β	a	b	Family	Author
–	–	1	1	W-G	Bourguignon et al. [44]
–	1	–	1	EE-G	Tahir et al. [70]
–	–	–	1	EW-G	Cordeiro et al. [71] and Hassan and Elgarhy [72]
1	2	–	1	BX-G	Yousof et al. [73]
2	–	–	1	TLW-G	Elbatal et al. [74]
$(1+\lambda^{\beta})$	–	1	1	WW-G	Hassan et al. [75]
–	2	1	1	R-G	–
–	1	1	1	E-G	–

6.2 Beta Weibull-G (BW-G) Family

The CDF of BW-G by Makubate et al. [76] is

$$F\left(x\right) = I_{\left(1-\exp\left\{-\alpha\left[\frac{G(x)}{\bar{G}(x)}\right]^{\lambda}\right\}\right)}\left(a,b\right).$$

Some structural properties and MLEs were reported. The BW-Uniform, BW-exponential, and BW-log-logistic were discussed. The last model was applied to real datasets, which proves that it outperforms some other distributions. This class encompasses many well-known models, some of them reported in Table 10.

Table 10: Special models of the BW-G Class

a	b	α	λ	Family	Author
1	1	–	–	W-G	Bourguignon et al. [44]
–	–	1	–	BW-G	Yousof et al. [77]
–	–	–	1	BE-G	–
–	–	–	2	BR-G	–
1	–	–	–	L-G	–
–	1	–	–	EW-G	Cordeiro et al. [71] and Hassan and Elgarhy [72]
1	–	–	2	ER-G	–
1	–	–	1	EE-G	–
1	1	–	2	R-G	–
1	1	–	1	E-G	–

6.3 Generalized Weibull Power Series (GWPS) Family

The CDF of GWPS by Rahmouni [78] is

$$F(x) = \sum_{n=k}^{\infty} \frac{a_n \theta^n}{C_k^L(\theta)} I_{[1-e^{-(\beta x)^\alpha}]}(k, n-k+1), \quad n \geq k,$$

where $C_k^L(\theta) = \sum_{n=k}^{\infty} a_n \theta^n$ is finite and $a_n > 0$ depends only on n.

The geometric, Poisson, binomial, and logarithmic were considered for the power series.

6.4 Discrete alpha power Weibull-G (DAPW-G) Family

The CDF of the DAPW-G by Balubaid and Alsulami [79] follows

$$F(x) = \frac{1}{\alpha - 1}\left[\alpha^{1-\exp\left(-\gamma\left\{\frac{-\log[1-G(x+1,\delta)]}{\beta}\right\}^c\right)} - 1\right], \text{ if } \alpha > 0, \alpha \neq 1$$

Analysis of four real datasets showed that the DAPWE distribution provides better results than other classes.

6.5 Exponentiated Power Generalized Weibull Power Series (EPGWPS) Family

The CDF of EPGWPS by Aldahlan et al. [80] is

$$F(x) = \frac{1}{C(\theta)} C\left(\theta\left\{1 - \exp\left[1 - (1 + \lambda x^\mu)^\alpha\right]\right\}^\beta\right)$$

The precision of the MLEs was proved in a simulation study. The usefulness of some sub-models was proved. Special cases are listed in Table 11.

Table 11: Some cases of the EPGWPS class

μ	α	β	Family	Author
–	1	1	WPS	Morais and Barreto-Souza [81]
–	1	–	EWPS	Mahmoudi and Jafari [82]
1	1	–	EExPS	Mahmoudi and Jafari [82]
1	1	1	ExPS	Mahmoudi and Jafari [82]
–	–	1	PGWPS	Nasiru and Abubakari [83]
2	1	–	BXPS	–
1	–	–	ENHPS	–
1	–	1	NHPS	–

7 Conclusion and Some Proposed Families

This paper presents a comprehensive survey of existing Weibull-based families, serving as a reference framework to assist practitioners in selecting suitable distributions and providing a foundation for researchers aiming to develop new extensions of the Weibull family. The widespread development of Weibull-based families can be theoretically justified by the inherent limitations of the classical Weibull model, whose failure rate is restricted to monotone shapes, despite its central role in reliability analysis and extreme-value theory. By embedding the Weibull distribution within a generator-based framework, these extensions achieve the necessary flexibility to model complex hazard behaviors while preserving the interpretability and tractability of the Weibull baseline. This generator representation therefore offers a unified methodological explanation for the rapid proliferation of Weibull-generated models in the literature.

It is important to mention that from the previous Weibull families, we can construct new classes by combining two of them. We provide some new classes from the previous generators:

- Applying the generator from Section 3.1 to the generator in Section 4.4, we can form the ExOW-W-G family with 3 additional parameters.
- The WMO-W-G family with 3 additional parameters follows by applying the generator from Section 3.1 to the generator from Section 4.6.
- The three-parameter ETIW-W-G family can be obtained by inserting the generator from Section 3.1 as a baseline model to the generator from Section 4.9.
- The four-parameter EW-W-G family follows by applying the generator in Section 3.1 to the generator in Section 5.2.
- Applying the generator from Section 4.1 to the generator from Section 4.2, we can form the MOEW-TW-G family with 4 additional parameters.

One of the main packages in R specifically designed to fit and test the adequacy of continuous distributions is the "AdequacyModel" package, which has a general-purpose optimization method (Particle Swarm Optimization) useful for maximizing complex likelihood functions and statistical measures such as Carmér-von Mises, Anderson Darling, and AIC, CAIC, BIC and HQIC statistics. This script provides functions to fit distributions and evaluate the goodness of fit using various metrics. It can be installed from Comprehensive R Archive Network (CRAN) and loaded into your R session. Then, the goodness.fit() function can be used given the data and the CDF and PDF of the chosen distribution. It is available at https://cran.r-project.org/web/packages/AdequacyModel/index.html

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No AI-based tools were used in the generation, writing, revising, analysis, or interpretation of the research data and findings presented in this article.

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